C3 Revision Questions

(using questions from January 2006, January 2007, January 2008 and January 2009)

1 Algebra and functions

What students need to learn:

Simplification of rational expressions including factorising and cancelling, and algebraic division.	Denominators of rational expressions will be linear or quadratic, eg $\frac{1}{ax+b}$, $\frac{ax+b}{px^2+qx+r}$, $\frac{x^3+1}{x^2-1}$.

1. $\mathrm{f}(x) = 1 - \dfrac{3}{x+2} + \dfrac{3}{(x+2)^2}$, $x \neq -2$.

(*a*) Show that $\mathrm{f}(x) = \dfrac{x^2 + x + 1}{(x+2)^2}$, $x \neq -2$. **(4)**

(*b*) Show that $x^2 + x + 1 > 0$ for all values of x. **(3)**

(*c*) Show that $\mathrm{f}(x) > 0$ for all values of x, $x \neq -2$. **(1)**

2. Express $\dfrac{2x^2 + 3x}{(2x+3)(x-2)} - \dfrac{6}{x^2 - x - 2}$ as a single fraction in its simplest form. **(7)**

3. Given that $\dfrac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \dfrac{dx + e}{(x^2 - 1)}$,

find the values of the constants a, b, c, d and e. **(4)**

4. $\mathrm{f}(x) = \dfrac{2x+2}{x^2-2x-3} - \dfrac{x+1}{x-3}$.

(*a*) Express f(*x*) as a single fraction in its simplest form. **(4)**

(*b*) Hence show that $\mathrm{f}'(x) = \dfrac{2}{(x-3)^2}$. **(3)**

Definition of a function. Domain and range of functions. Composition of functions. Inverse functions and their graphs.

The concept of a function as a one-one or many-one mapping from $\mathbb{R}$ (or a subset of $\mathbb{R}$) to $\mathbb{R}$. The notation $\mathrm{f} : x \mapsto$ and $\mathrm{f}(x)$ will be used.

Students should know that fg will mean 'do g first, then f'.

Students should know that if f^{-1} exists, then $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{f}\mathrm{f}^{-1}(x) = x$.

5. The functions f and g are defined by $\mathrm{f}: x \mapsto 2x + \ln 2, \quad x \in \mathbb{R},$

$$\mathrm{g}: x \mapsto \mathrm{e}^{2x}, \quad x \in \mathbb{R}.$$

(*a*) Prove that the composite function gf is $\mathrm{gf}: x \mapsto 4\mathrm{e}^{4x}, \quad x \in \mathbb{R}.$ **(4)**

(*b*) Sketch the curve with equation $y = \mathrm{gf}(x)$, and show the coordinates of the point where the curve cuts the y-axis. **(1)**

(*c*) Write down the range of gf. **(1)**

(*d*) Find the value of x for which $\frac{\mathrm{d}}{\mathrm{d}x}[\mathrm{gf}(x)] = 3$,

giving your answer to 3 significant figures. **(4)**

6. The functions f and g are defined by $f : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R},$

$$g : x \mapsto e^{x^2}, \quad x \in \mathbb{R}.$$

(*a*) Write down the range of g. **(1)**

(*b*) Show that the composite function fg is defined by $fg : x \mapsto x^2 + 3e^{x^2}, \quad x \in \mathbb{R}.$ **(2)**

(*c*) Write down the range of fg. **(1)**

(*d*) Solve the equation $\frac{d}{dx}\left[fg(x)\right] = x(xe^{x^2} + 2).$ **(6)**

7. The function f is defined by $\mathrm{f} : x \mapsto \ln(4 - 2x)$, $x < 2$ and $x \in \mathbb{R}$.

(*a*) Show that the inverse function of f is defined by $\mathrm{f}^{-1} : x \mapsto 2 - \frac{1}{2}\mathrm{e}^{x}$ and write down the domain of f^{-1}. **(4)**

(*b*) Write down the range of f^{-1}. **(1)**

(*c*) Sketch the graph of $y = \mathrm{f}^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes. **(4)**

The graph of $y = x + 2$ crosses the graph of $y = \mathrm{f}^{-1}(x)$ at $x = k$.

The iterative formula $x_{n+1} = -\frac{1}{2}\mathrm{e}^{x_n}$, $x_0 = -0.3$,is used to find an approximate value for k.

(*d*) Calculate the values of x_1 and x_2, giving your answer to 4 decimal places. **(2)**

(*e*) Find the values of k to 3 decimal places. **(2)**

8. The functions f and g are defined by $f : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}.$

$$g : x \mapsto \frac{3}{x} - 4, \quad x > 0, \; x \in \mathbb{R}.$$

(*a*) Find the inverse function f^{-1}. **(2)**

(*b*) Show that the composite function gf is $gf : x \mapsto \dfrac{8x^3 - 1}{1 - 2x^3}$. **(4)**

(*c*) Solve gf $(x) = 0$. **(2)**

(*d*) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$. **(5)**

The modulus function.	Students should be able to sketch the graphs of $y = \lvert ax + b \rvert$ and the graphs of $y = \lvert f(x) \rvert$ and $y = f(\lvert x \rvert)$, given the graph of $y = f(x)$.
Combinations of the transformations $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.	Students should be able to sketch the graph of, for example, $y = 2f(3x)$, $y = f(-x) + 1$, given the graph of $y = f(x)$ or the graph of, for example, $y = 3 + \sin 2x$, $y = -\cos\left(x + \frac{\pi}{4}\right)$. The graph of $y = f(ax + b)$ will *not* be required.

9.

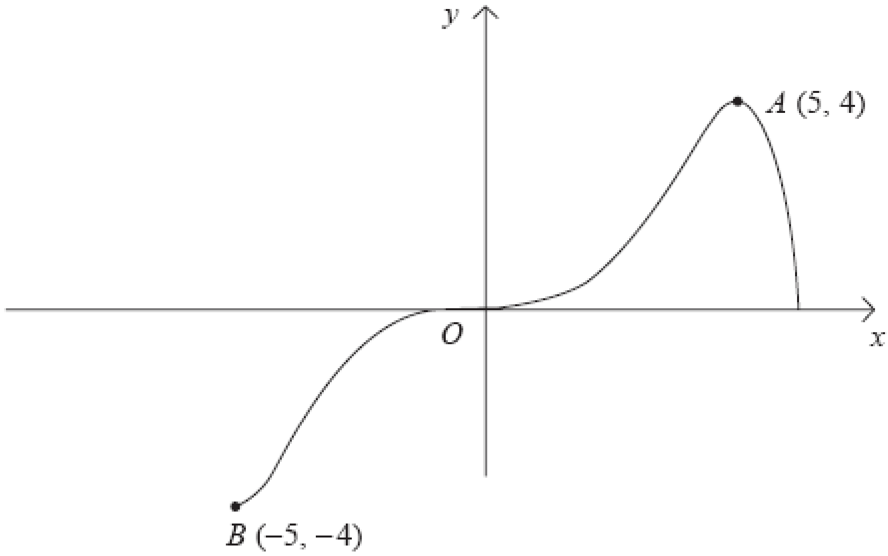


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \text{f}(x)$.
The curve passes through the origin *O* and the points *A*(5, 4) and *B*(–5, –4).
In separate diagrams, sketch the graph with equation

(*a*) $y = |\text{f}(x)|$, **(3)**

(*b*) $y = \text{f}(|x|)$, **(3)**

(*c*) $y = 2\text{f}(x + 1)$. **(4)**

On each sketch, show the coordinates of the points corresponding to *A* and *B*.

10. **Figure 1**

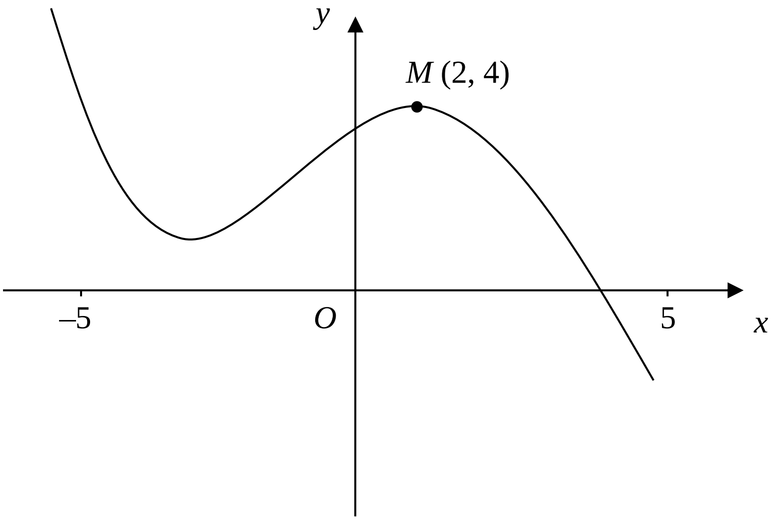


Figure 1 shows the graph of $y = \mathrm{f}(x)$, $-5 \le x \le 5$.

The point M (2, 4) is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(*a*) $y = \mathrm{f}(x) + 3$, **(2)**

(*b*) $y = |\mathrm{f}(x)|$, **(2)**

(*c*) $y = \mathrm{f}(|x|)$. **(3)**

Show on each graph the coordinates of any maximum turning points.

11. $$f(x) = x^4 - 4x - 8.$$

(*a*) Show that there is a root of $f(x) = 0$ in the interval $[-2, -1]$. **(3)**

(*b*) Find the coordinates of the turning point on the graph of $y = f(x)$. **(3)**

(*c*) Given that $f(x) = (x-2)(x^3 + ax^2 + bx + c)$, find the values of the constants a, b and c. **(3)**

(*d*) Sketch the graph of $y = f(x)$. **(3)**

(*e*) Hence sketch the graph of $y = |f(x)|$. **(1)**

12.

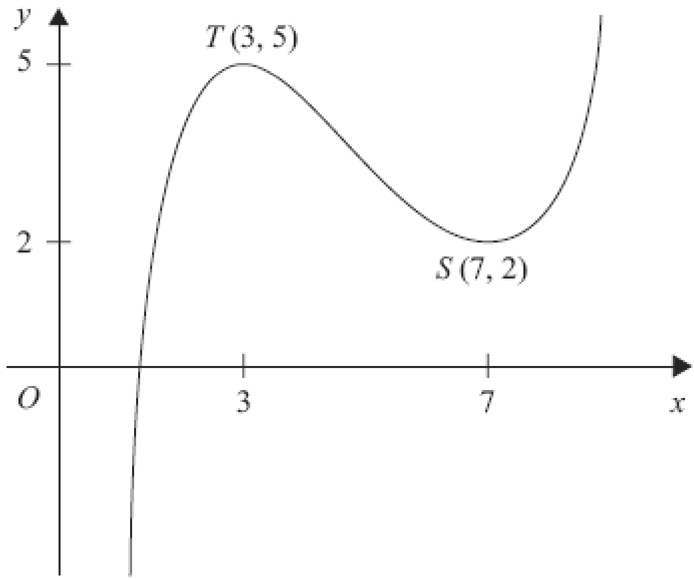


Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $1 < x < 9$.

The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(*a*) $y = 2\mathrm{f}(x) - 4$, **(3)**

(*b*) $y = |\mathrm{f}(x)|$. **(3)**

Indicate on each diagram the coordinates of any turning points on your sketch.

2 Trigonometry

What students need to learn:

Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains.	Angles measured in both degrees and radians.
Knowledge and use of $\sec^2\theta = 1 + \tan^2\theta$ and $\text{cosec}^2\theta = 1 + \cot^2\theta$.	
Knowledge and use of double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ and of expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$.	To include application to half angles. Knowledge of the t ($\tan\frac{1}{2}\theta$) formulae will *not* be required. Students should be able to solve equations such as $a\cos\theta + b\sin\theta = c$ in a given interval, and to prove simple identities such as $\cos x\cos 2x + \sin x\sin 2x \equiv \cos x$.

13. (*a*) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. **(5)**

(*b*) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$. **(2)**

14. (*a*) Show that

(i) $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \qquad x \neq (n - \tfrac{1}{4})\pi,\ n \in \mathbb{Z},$ **(2)**

(ii) $\tfrac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \tfrac{1}{2}$. **(3)**

(*b*) Hence, or otherwise, show that the equation $\cos\theta\left(\dfrac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \dfrac{1}{2}$ can be written as

$\sin 2\theta = \cos 2\theta$. **(3)**

(*c*) Solve, for $0 \leq \theta < 2\pi$, $\sin 2\theta = \cos 2\theta$, giving your answers in terms of π. **(4)**

15. (*a*) (i) By writing $3\theta = (2\theta + \theta)$, show that $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$. **(4)**

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve $8 \sin^3 \theta - 6 \sin \theta + 1 = 0$.

Give your answers in terms of π. **(5)**

(*b*) Using $\sin (\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$, or otherwise, show that

$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2})$. **(4)**

16. (*a*) Use the double angle formulae and the identity

$$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only. **(4)**

(*b*) (i) Prove that $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} \equiv 2 \sec x, \qquad x \neq (2n + 1)\dfrac{\pi}{2}$. **(4)**

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = 4$. **(3)**

17. (i) Prove that $\sec^2 x - \text{cosec}^2 x \equiv \tan^2 x - \cot^2 x$. **(3)**

(ii) Given that $y = \arccos x$, $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$,

(*a*) express $\arcsin x$ in terms of y. **(2)**

(*b*) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of π. **(1)**

18. (*a*) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos (\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. **(4)**

(*b*) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs. **(3)**

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos (15t)° + 4 \sin (15t)°,$$

where t is the time in hours from midday and $0 \leq t < 24$.

(*c*) Calculate the minimum temperature of the warehouse as given by this model. **(2)**

(*d*) Find the value of t when this minimum temperature occurs. **(3)**

19. $f(x) = 12\cos x - 4\sin x$.

Given that $f(x) = R\cos(x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90°$,

(*a*) find the value of R and the value of α. **(4)**

(*b*) Hence solve the equation $12\cos x - 4\sin x = 7$ for $0 \leq x < 360°$, giving your answers to one decimal place. **(5)**

(*c*) (i) Write down the minimum value of $12\cos x - 4\sin x$. **(1)**

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs. **(2)**

3 Exponentials and logarithms

What students need to learn:

The function e^x and its graph.	To include the graph of $y = e^{ax+b} + c$.
The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x.	Solution of equations of the form $e^{ax+b} = p$ and $\ln(ax + b) = q$ is expected.

20. The radioactive decay of a substance is given by $R = 1000e^{-ct}$, $t \geq 0$.
where R is the number of atoms at time t years and c is a positive constant.

(*a*) Find the number of atoms when the substance started to decay. **(1)**

It takes 5730 years for half of the substance to decay.

(*b*) Find the value of c to 3 significant figures. **(4)**

(*c*) Calculate the number of atoms that will be left when $t = 22\,920$. **(2)**

(*d*) Sketch the graph of R against t . **(2)**

4 Differentiation

What students need to learn:

Differentiation of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$ and their sums and differences.	
Differentiation using the product rule, the quotient rule and the chain rule.	Differentiation of $\operatorname{cosec} x$, $\cot x$ and $\sec x$ are required. Skill will be expected in the differentiation of functions generated from standard forms using products, quotients and composition, such as $2x^4 \sin x$, $\frac{e^{3x}}{x}$, $\cos x^2$ and $\tan^2 2x$.
The use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$.	Eg finding $\frac{dy}{dx}$ for $x = \sin 3y$.

21. (i) The curve C has equation $y = \dfrac{x}{9 + x^2}$.

Use calculus to find the coordinates of the turning points of C. **(6)**

(ii) Given that $y = (1 + e^{2x})^{\frac{3}{2}}$, find the value of $\dfrac{dy}{dx}$ at $x = \frac{1}{2}\ln 3$. **(5)**

22. (*a*) Differentiate with respect to x

(i) x^2e^{3x+2}, **(4)**

(ii) $\dfrac{\cos(2x^3)}{3x}$. **(4)**

(*b*) Given that $x = 4\sin(2y + 6)$, find $\dfrac{dy}{dx}$ in terms of x. **(5)**

23. (*a*) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation $y = x^2 \sqrt{(5x - 1)}$.

(6)

(*b*) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x. **(4)**

24. A curve C has equation $y = 3 \sin 2x + 4 \cos 2x, \quad -\pi \leq x \leq \pi$.
The point $A(0, 4)$ lies on C.

(*a*) Find an equation of the normal to the curve C at A. **(5)**

(*b*) Express y in the form $R \sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

Give the value of α to 3 significant figures. **(4)**

(*c*) Find the coordinates of the points of intersection of the curve C with the x-axis.
Give your answers to 2 decimal places. **(4)**

25. The curve C has equation $x = 2 \sin y$.

(*a*) Show that the point $P\left(\sqrt{2},\ \frac{\pi}{4}\right)$ lies on C. **(1)**

(*b*) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P. **(4)**

(*c*) Find an equation of the normal to C at P. Give your answer in the form $y = mx + c$, where m and c are exact constants. **(4)**

26. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found. **(6)**

27. A curve C has equation $y = e^{2x} \tan x, \quad x \neq (2n + 1)\frac{\pi}{2}$.

(a) Show that the turning points on C occur where $\tan x = -1$. **(6)**

(b) Find an equation of the tangent to C at the point where $x = 0$. **(2)**

28. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants. **(5)**

5 Numerical methods

What students need to learn:

Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous.	
Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$.	Solution of equations by use of iterative procedures for which leads will be given.

29. $f(x) = 2x^3 - x - 4$.

(*a*) Show that the equation $f(x) = 0$ can be written as $x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}$. **(3)**

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(*b*) Use the iteration formula $x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)}$, with $x_0 = 1.35$, to find, to 2 decimal places, the value of x_1, x_2 and x_3. **(3)**

The only real root of $f(x) = 0$ is α.

(*c*) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places. **(3)**

30. $\mathrm{f}(x) = 3x\mathrm{e}^x - 1$.

The curve with equation $y = \mathrm{f}(x)$ has a turning point P.

(*a*) Find the exact coordinates of P. **(5)**

The equation $\mathrm{f}(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$.

(*b*) Use the iterative formula $x_{n+1} = \frac{1}{3}\mathrm{e}^{-x_n}$, with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3. **(3)**

(*c*) By choosing a suitable interval, show that a root of $\mathrm{f}(x) = 0$ is $x = 0.2576$ correct to 4 decimal places. **(3)**

31. $f(x) = \ln(x + 2) - x + 1, \qquad x > -2, x \in \mathbb{R}$.

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$. **(2)**

(b) Use the iterative formula $x_{n+1} = \ln(x_n + 2) + 1$, $x_0 = 2.5$, to calculate the values of x_1, x_2 and x_3, giving your answers to 5 decimal places. **(3)**

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places. **(2)**

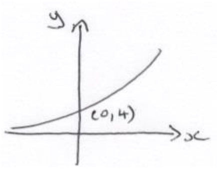
Answers

2) $\frac{x+3}{x=1}$

3) a=2, b=0, c=-1, d=1, e=0

4) $\frac{1-x}{x-3}$

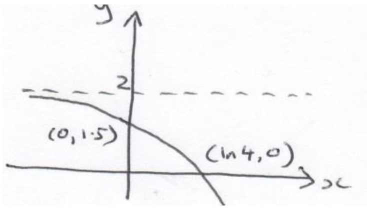
5)b)



5)c) gf(x) > 0   d) -0.418

6) a) g(x) ≥ 1   c) fg(x) ≥ 3   d) 0 or 6

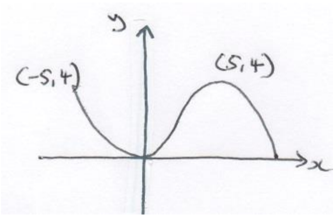
7) a) x∈ R   b) $f^{-1}(x) < 2$ 7)c)



d) x_1 = -0.3704, x_2 = -0.3452 e) k ≈ -0.352

8) a) f-1(x) = $\left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ c) x = ½   d) (0,-1)

9 a)



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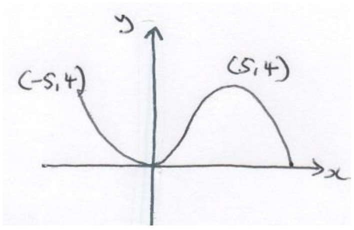

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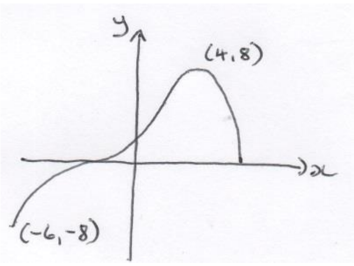

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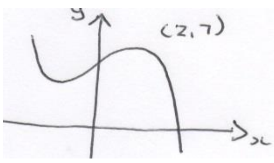

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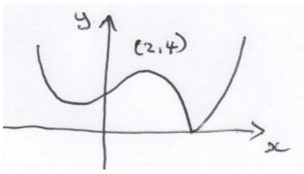

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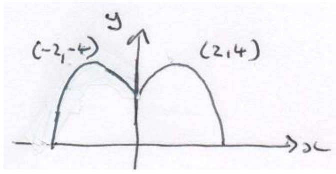
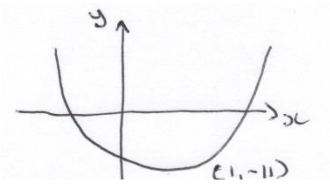


11) b) (1,-11)   c) a=2, b=4, c=4   d)



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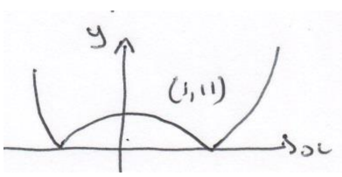
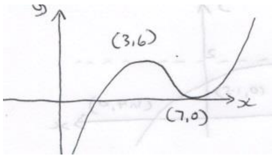
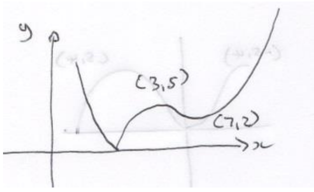


12) a) 

b) 

13) b) $\frac{9\sqrt{3}}{16}$

14) c) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

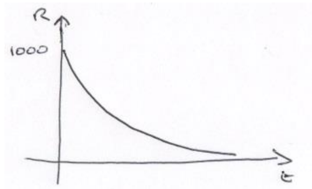
15) a) ii) $\frac{\pi}{18}, \frac{5\pi}{18}$

16) a) $4\cos^3 x - 3\cos x$ b) ii) $\frac{\pi}{3}, \frac{5\pi}{3}$

17) ii) a) $\frac{\pi}{2} - y$ b) $\frac{\pi}{2}$

18) a) $5\cos(\theta - 53°)$ b) 5, 53° c) 5° d) 15.5

19) a) $R = \sqrt{160}$, $\alpha = 18.43°$ b) 38.0°, 285.2° c) i) $-\sqrt{160}$ ii) 161.57°

20) a) 1000 b) 0.000121 c) 62.5 d) 

21) i) $(3, \frac{1}{6})$ and $(-3, -\frac{1}{6})$ ii) 18

22) a) i) $3x^2 e^{3x+2} + 2x e^{3x+2}$ ii) $\frac{-18x^3 \sin(2x^3) - 3\cos 2x^3}{9x^2}$ b) $\pm \frac{1}{2\sqrt{16 - x^2}}$

23) a) $\frac{46}{3}$ b) $\frac{2x\cos 2x - 2\sin 2x}{x^3}$

24 a) $y - 4 = -\frac{1}{6}x$ b) $5\sin(2x + 0.927)$ c) -2.03, -0.46, 1.11, 2.68

25) c) $y = -\sqrt{2}\,x + 2 + \frac{\pi}{4}$

26) $y = \frac{1}{2}x + \frac{\pi}{4}$

27) b) $y = x$

28) $y = -9x + 27$

29) b) 1.41, 1.39, 1.39

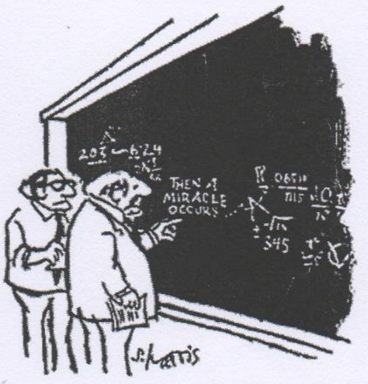
30) a) $(-1, -3e^{-1} - 1)$ b) 0.2596, 0.2571, 0.2578

31) b) $x_1 = \ln 4.5 + 1 \approx 2.50408$, $x_2 \approx 2.50498$, $x_3 \approx 2.50518$

BHASVIC
MαTHS

C3 Survival Kit

Core 3



"I think you should be more explicit here in step two."

1

Algebraic Fractions

Typical expensive errors include:

- Expanding brackets wrongly

  e.g. $5 - 3(x + y) \neq 5 - 3x + 3y$

- Massively overcomplicating the algebra

- Letting a minus sign defeat you

- Using brackets incorrectly

- Cancelling things illegally by just crossing them out

  $\frac{x^2+y}{p^2+y} \neq \frac{x^2}{p^2}$

Natural Logs

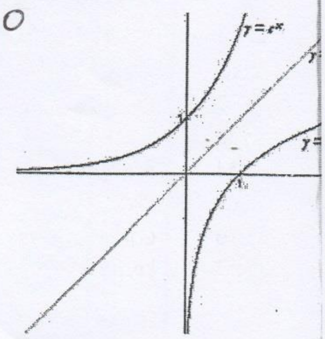
$e = 2.718...$ $\quad \log_e x = \ln x$ $\quad \ln e = 1$ $\quad \ln 1 = 0$

Usual laws of logs apply:

$\ln ab = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln x^n = n \ln x$



$\ln 3t \neq \ln 3t$

Derivatives to learn

Function	Derivative	Function	Derivative
sin x	$\cos x$	sec x	$\sec x \tan x$
cos x	$-\sin x$	cosec x	$-\operatorname{cosec} x \cot x$
tan x	$\sec^2 x$	cot x	$-\operatorname{cosec}^2 x$
e^x	e^x	ln x	$\frac{1}{x}$
x^n	nx^{n-1}	a^x (C4 only)	$a^x \ln a$

Chain Rule f[g(x)]

$$\frac{d}{dx}\left(f[g(x)]\right) = f'[g(x)]\, g'(x)$$

Product Rule

$$\frac{d}{dx}\left(f(x)\,g(x)\right) = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

SCAN FOR YOUTUBE VIDEOS

Chain Rule

Product Rule

Quotient Rule

3

Oooo lovely: Trig Formulae

$\cos^2 A + \sin^2 A = 1$ $\quad$ $1 + \cot^2 A = \operatorname{cosec}^2 A$ $\quad$ $1 + \tan^2 A = \sec^2 A$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\qquad$ $\tan x = \dfrac{\sin x}{\cos x}$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin 2A = 2 \sin A \cos A$ $\qquad$ $\cos 2A = 2\cos^2 A - 1$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$ $\qquad$ $\cos 2A = 1 - 2\sin^2 A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$ $\qquad$ $\cos^2 A = \frac{1}{2} + \frac{1}{2}\cos 2A$

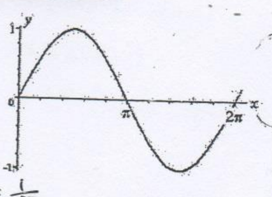
$\sin x + 2\cos x = R\sin(x+a)$ $\quad$ $\sqrt{1^2+2^2}$

$= \sqrt{5}\left(\frac{1}{\sqrt{5}}\sin x + \frac{2}{\sqrt{5}}\cos x\right) = R\sin(x+\alpha)$

$= \sqrt{5}(\sin x \cos\alpha + \cos x \sin\alpha) = R\sin(x+\alpha)$

$= \sqrt{5}\sin(x + 63.4) = R\sin(x+\alpha)$

$= \quad R = \sqrt{5}, \quad \alpha = 63.4°$



$\cos\alpha = \frac{1}{\sqrt{5}}$

$\sin\alpha = \frac{2}{\sqrt{5}}$ $\qquad$ $\alpha = 63.4°$

For $R\sin(x+a)$ the maximum, R, is achieved when $x+a =$ 90, 450

For $R\cos(x+a)$ the maximum, R, is achieved when $x+a =$ 0, 360

$\sin^2 A = 1 - \cos^2 A$

Using recurrence relations to find approximate roots

$f(x) = \ldots$ is continuous and changes sign between $x = a$ and $x = b$, therefore $f(x)$ has a root between $x = a$ and $x = b$. This proves that the root is $x = \ldots$ to 2* dp

$x^2 - 5x = 3$ has a root in the interval $5 < x < 6$.
Show that the root of this equation also solves the equation $x = \sqrt{5x+3}$

$x^2 - 5x = 3$
$\therefore x^2 = 5x + 3$
$\therefore x = \sqrt{5x+3}$

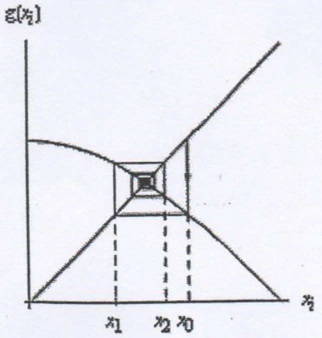
Take $x_0 = 5$ and use iteration to find 4 improved approximations to the root of the equation $x^2 - 5x = 3$

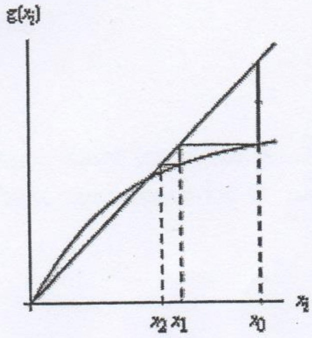
$x_0 = 5$
$x_1 = \sqrt{5x_0+3} = \sqrt{28} = 5.292$
$x_2 = \sqrt{5x_1+3} = \sqrt{29.4575} = 5.427$
$x_3 = \sqrt{5x_2+3} = \sqrt{30.1374} = 5.490$
$x_4 = \sqrt{5x_3+3} = \sqrt{30.4488} = 5.518$

Prove that the root is 5.5 to 1 d.p.

Let $f(x) = x^2 - 5x - 3$
$f(5.55) = 0.0525$
$f(5.45) = -0.5475$

Because there is a change of sign and because $f(x)$ is continuous, a root of the equation is 5.5





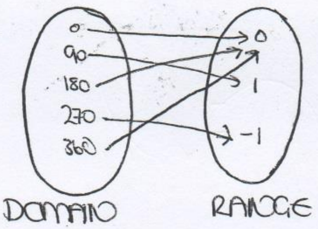
use this to get to the curve ↕

use this to get to y = x ⟷

Range & Domain, Inverse, Composites

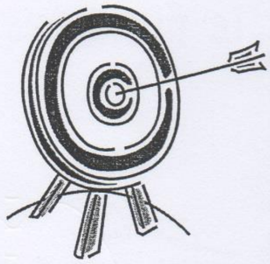
Domain and Range

$f(x) = \sin x$



Domain is the starting numbers

Range is the ending numbers



This is an example of a many-one function.

Inverse

$f(x) = 5x - 2$

$f^{-1}(x) = \frac{x+2}{5}$

going from the range numbers to the domain numbers

Composite

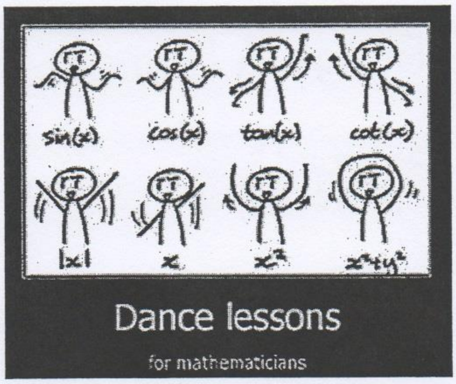
$f(x) = x^2$ | $fg(x) =$ do g first $= (2x+3)^2$

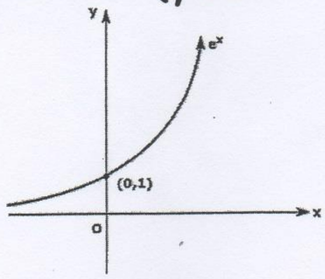
$g(x) = 2x+3$ | $gf(x) =$ do f first $= 2x^2 + 3$

Function

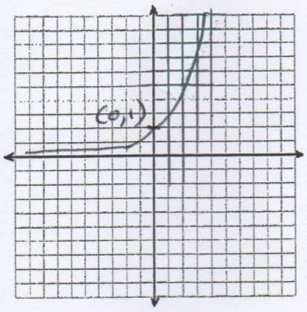
A function is a mapping such that every element of the domain is mapped to exactly one element of the range.

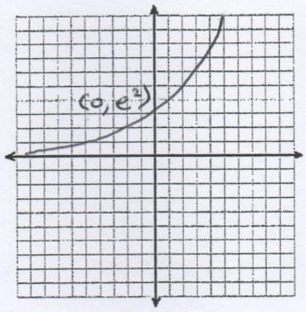
Graph sketching is fun (yes it is)

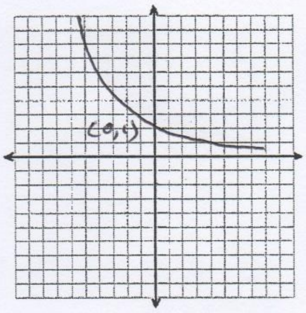




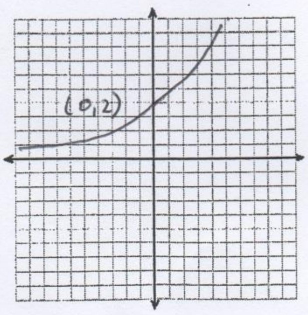
$f(x) = e^x$

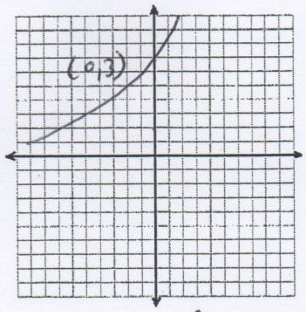


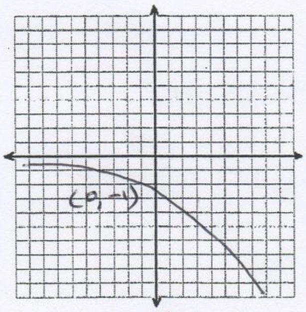




$f(2x)$ ↗ $= e^{2x}$ $f(x+2)$ ↗ $= e^{x+2}$ $f(-x)$ ↗ $= e^{-x}$







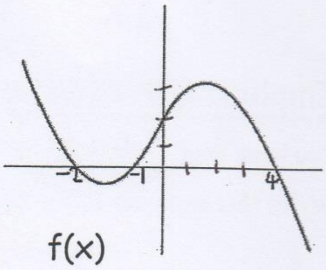
$2f(x)$ ↗ $= 2e^x$ $f(x) + 2$ ↗ $= e^x + 2$ $-f(x)$ ↗ $= -e^x$

Remember to show the x-intercept: and the y-intercept:

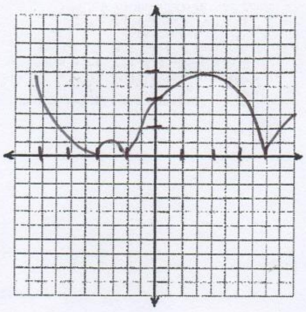
Now you can apply combinations of these transformations to e^x and lnx
Always label the equation of the asymptote!

7

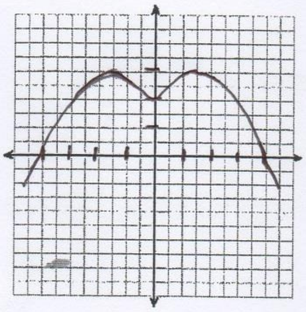
Modulus Equations need to be solved using a <u>graph</u> so you know which equations to put together!



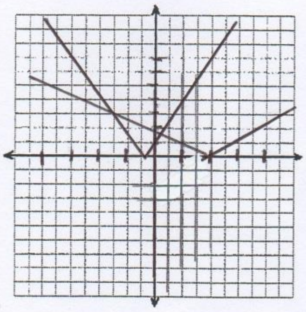
f(x)



|f(x)|



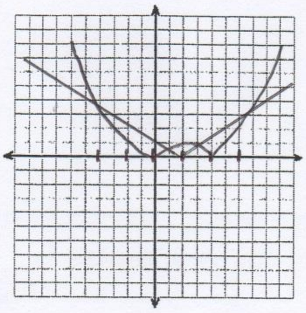
f(|x|)



$|3x + 1| = |x - 2|$

top tips...

Exaggerate the steepness of the steeper line to make sure you get all the intercepts



$|x(x-2)| = |x-1|$

8

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

f(*x*)	**f′(*x*)**
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$