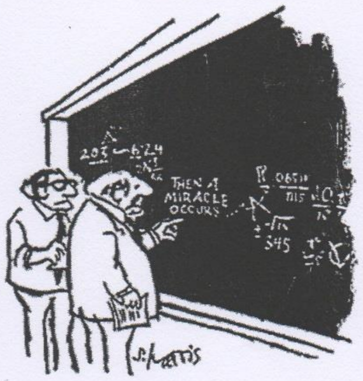
BHASVIC
mαths

C3 C4 S2 Survival Kit



"I think you should be more explicit here in step two."

Name ....

Algebraic Fractions

Typical expensive errors include:

- Expanding brackets wrongly

  e.g. $5 - 3(x + y) \neq 5 - 3x + 3y$

- Massively overcomplicating the algebra

- Letting a minus sign defeat you

- Using brackets incorrectly

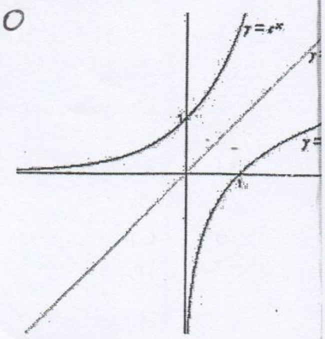
- Cancelling things illegally by just crossing them out

  $\frac{x^2+y}{p^2+y} \neq \frac{x^2}{p^2}$

Natural Logs

$e = 2.718\ldots$ $\quad \log_e x = \ln x \quad \ln e = 1 \quad \ln 1 = 0$

Usual laws of logs apply:



$\ln ab = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln x^n = n \ln x$

$\ln 3t \neq \ln 3 t$

Derivatives to learn

Function	Derivative	Function	Derivative
sin x	$\cos x$	sec x	$\sec x \tan x$
cos x	$-\sin x$	cosec x	$-\operatorname{cosec} x \cot x$
tan x	$\sec^2 x$	cot x	$-\operatorname{cosec}^2 x$
e^x	e^x	ln x	$\frac{1}{x}$
x^n	nx^{n-1}	a^x (C4 only)	$a^x \ln a$

Chain Rule f[g(x)]

$$\frac{d}{dx}\left(f[g(x)]\right) = f'[g(x)]\,g'(x)$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Product Rule

$$\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

SCAN FOR YOUTUBE VIDEOS

Chain Rule

Product Rule

Quotient Rule

Oooo lovely: Trig Formulae

$\cos^2 A + \sin^2 A = 1$ $\quad 1 + \cot^2 A = \operatorname{cosec}^2 A$ $\quad 1 + \tan^2 A = \sec^2 A$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\qquad \tan x = \dfrac{\sin x}{\cos x}$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin 2A = 2 \sin A \cos A$ $\qquad \cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

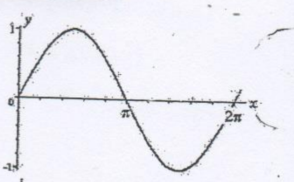
$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A$

$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$ $\qquad \cos^2 A = \frac{1}{2} + \frac{1}{2}\cos 2A$

$\sin x + 2\cos x = R\sin(x+a)$

$\sqrt{1^2 + 2^2}$



$= \sqrt{5}\left(\frac{1}{\sqrt{5}}\sin x + \frac{2}{\sqrt{5}}\cos x\right) = R\sin(x + \alpha)$

$= \sqrt{5}(\sin x \cos \alpha + \cos x \sin \alpha) = R\sin(x + \alpha)$

$= \sqrt{5}\sin(x + 63{\cdot}4) = R\sin(x+\alpha)$ $\qquad \cos\alpha = \frac{1}{\sqrt{5}}$

$= \quad R = \sqrt{5}, \quad \alpha = 63{\cdot}4^\circ$ $\qquad \sin\alpha = \frac{2}{\sqrt{5}}$ $\qquad \alpha = 63{\cdot}4^\circ$

For Rsin(x+a) the maximum, R, is achieved when x+a= 90, 450

For Rcos(x+a) the maximum, R, is achieved when x+a= 0, 360

$\sin^2 A = 1 - \cos^2 A$

Using recurrence relations to find approximate roots

$f(x) = \ldots$ is continuous and changes sign between $x = a$ and $x = b$, therefore $f(x)$ has a root between $x = a$ and $x = b$. This proves that the root is $x = \ldots$ to 2* dp

$x^2 - 5x = 3$ has a root in the interval $5 < x < 6$.

Show that the root of this equation also solves the equation $x = \sqrt{5x+3}$

$x^2 - 5x = 3$

$\therefore\ x^2 = 5x + 3$

$\therefore\ x = \sqrt{5x+3}$

Take $x_0 = 5$ and use iteration to find 4 improved approximations to the root of the equation $x^2 - 5x = 3$

$x_0 = 5$

$x_1 = \sqrt{5x_0+3} = \sqrt{28} = 5.292$

$x_2 = \sqrt{5x_1+3} = \sqrt{29.4575} = 5.427$

$x_3 = \sqrt{5x_2+3} = \sqrt{30.1374} = 5.490$

$x_4 = \sqrt{5x_3+3} = \sqrt{30.4488} = 5.518$

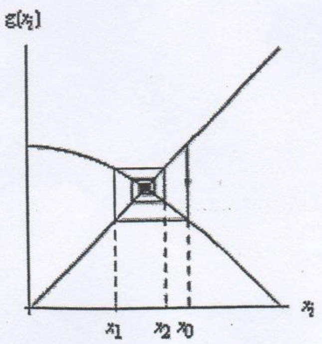
Prove that the root is 5.5 to 1 d.p.

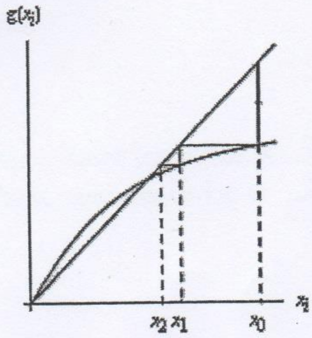
Let $f(x) = x^2 - 5x - 3$

$f(5.55) = 0.0525$

$f(5.45) = -0.5475$

Because there is a change of sign and because $f(x)$ is continuous, a root of the equation is 5.5





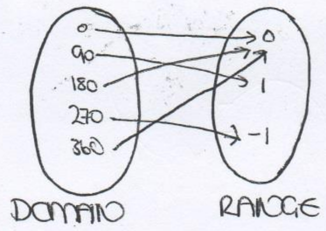
use this to get to the curve ↕

use this to get to y = x ↔

Range & Domain, Inverse, Composites

Domain and Range

$f(x) = \sin x$



Domain is the starting numbers

Range is the ending numbers

This is an example of a many-one function.

Inverse

$f(x) = 5x - 2$

$f^{-1}(x) = \dfrac{x+2}{5}$

going from the range numbers to the domain numbers

Composite

$f(x) = x^2$

$g(x) = 2x+3$

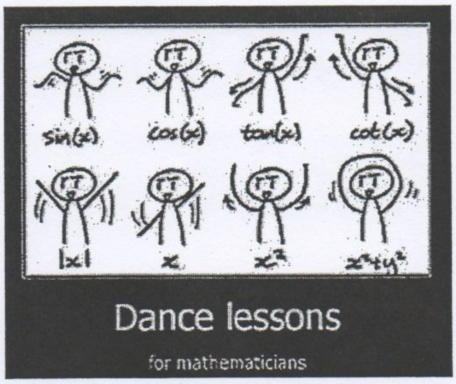
$fg(x) =$ do g first $= (2x+3)^2$

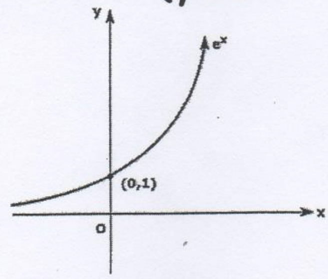
$gf(x) =$ do f first $= 2x^2 + 3$

Function

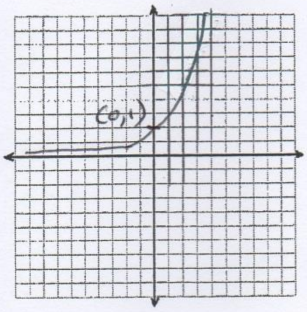
A function is a mapping such that every element of the domain is mapped to exactly one element of the range.

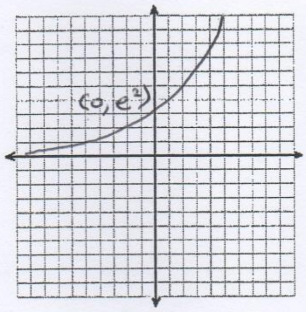
Graph sketching is fun (yes it is)

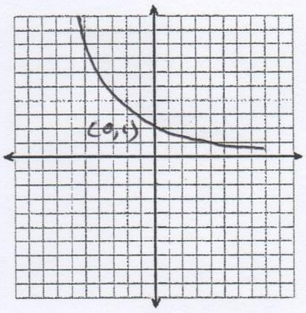




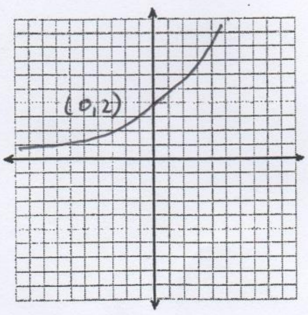
$f(x) = e^x$

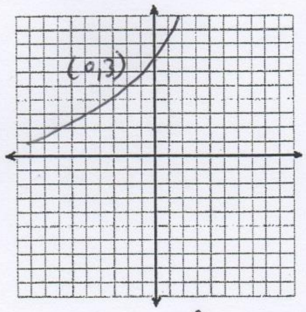


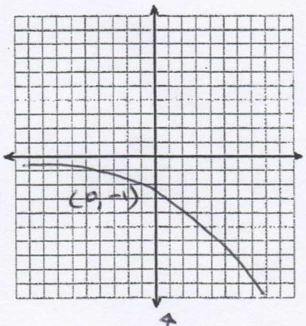




$f(2x) = e^{2x}$ $\quad$ $f(x+2) = e^{x+2}$ $\quad$ $f(-x) = e^{-x}$





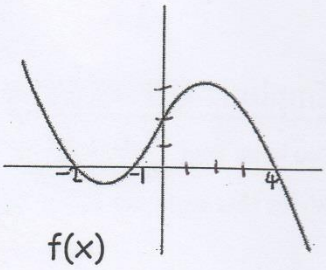


$2f(x) = 2e^x$ $\quad$ $f(x) + 2 = e^x + 2$ $\quad$ $-f(x) = -e^x$

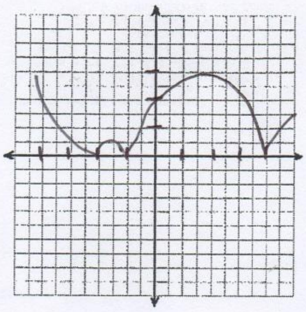
Remember to show the x-intercept: and the y-intercept:

Now you can apply combinations of these transformations to e^x and lnx
Always label the equation of the asymptote!

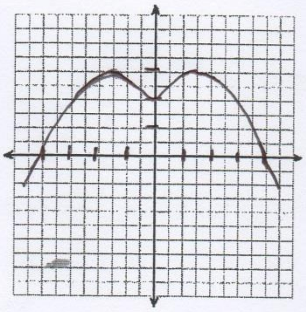
Modulus Equations need to be solved using a graph so you know which equations to put together!



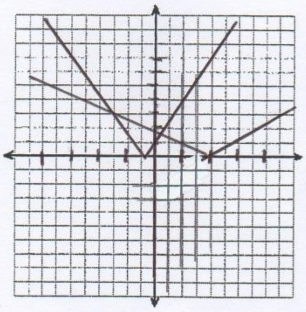
f(x)



|f(x)|



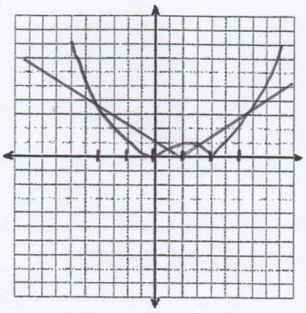
f(|x|)



$|3x + 1| = |x - 2|$

top tips...

Exaggerate the steepness of the steeper line to make sure you get all the intercepts



$|x(x-2)| = |x-1|$