

C3 C4 S2 Survival Kit

Core 3

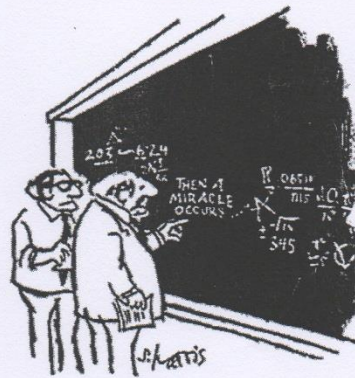
- Pg2 Algebraic Fractions
- Pg2 Natural Logs
- Pg3 Derivatives
- Pg4 Trig Formulae & $R\cos(x+a)$ for finding max/min
- Pg5 Using recurrence relations to find approximate roots to $f(x)=0$
- Pg6 Range & Domain, Inverse, Composites
- Pg7 Graph sketching
- Pg8 Modulus equations (solve using a graph)

Core 4

- Pg9 Implicit Differentiation
- Pg9 Partial Fractions
- Pg10-14 Integration
- Pg 15 Trapezium Rule
- Pg16 Binomial Expansion
- Pg17 Differential Equations
- Pg17 Connected Rates of Change
- Pg18 Parametric Equations
- Pg19 Vectors

Statistics 2

- Pg21 Accuracy
- Pg22 Binomial Distribution
- Pg23 Poisson Distribution
- Pg24 Approximation Triangle
- Pg25 Continuous Random Variables
- Pg26 Continuous Distribution
- Pg27 Hypothesis Testing



"I think you should be more explicit here in step two."

Name

Algebraic Fractions



Typical expensive errors include:

- Expanding brackets wrongly
e.g. $5 - 3(x + y) \neq 5 - 3x + 3y$

- Massively overcomplicating the algebra

- Letting a minus sign defeat you

- Using brackets incorrectly

- Cancelling things illegally by just crossing them out

$$\frac{x^2+y}{p^2+y} \neq \frac{x^2}{p^2}$$

Natural Logs

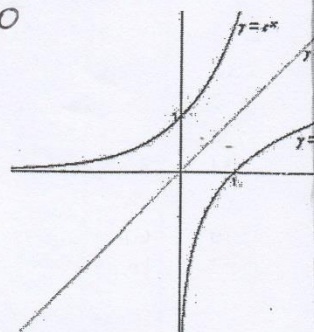
$$e = 2.718... \quad \log_e x = \ln x \quad \ln e = 1 \quad \ln 1 = 0$$

Usual laws of logs apply:

$$\ln ab = \ln a + \ln b$$

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

$$\ln x^n = n \ln x$$



$$\ln 3t \neq \ln 3t$$

Derivatives to learn

Function	Derivative	Function	Derivative
$\sin x$	$\cos x$	$\sec x$	$\sec x \tan x$
$\cos x$	$-\sin x$	$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\tan x$	$\sec^2 x$	$\cot x$	$-\operatorname{cosec}^2 x$
e^x	e^x	$\ln x$	$\frac{1}{x}$
x^n	nx^{n-1}	a^x (C4 only)	$a^x \ln a$
Chain Rule $f[g(x)]$ $\frac{d}{dx} (f[g(x)]) = f'[g(x)] g'(x)$		Quotient Rule $\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$	
Product Rule $\frac{d}{dx} (f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$		$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$	

SCAN FOR YOUTUBE VIDEOS

Chain Rule



Product Rule



Quotient Rule



Oooo lovely: Trig Formulae

$$\cos^2 A + \sin^2 A = 1 \quad 1 + \cot^2 A = \operatorname{cosec}^2 A \quad 1 + \tan^2 A = \sec^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$$

$$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$$

$$\sin x + 2 \cos x = R \sin(x + \alpha)$$

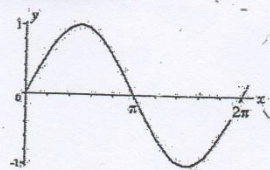
$$= \sqrt{5} \left(\frac{1}{\sqrt{5}} \sin x + \frac{2}{\sqrt{5}} \cos x \right) = R \sin(x + \alpha)$$

$$= \sqrt{5} (\sin x \cos \alpha + \cos x \sin \alpha) = R \sin(x + \alpha)$$

$$= \sqrt{5} \sin(x + 63.4^\circ) = R \sin(x + \alpha)$$

$$= R = \sqrt{5}, \quad \alpha = 63.4^\circ$$

$$\sqrt{1^2 + 2^2}$$



$$\cos \alpha = \frac{1}{\sqrt{5}}$$

$$\sin \alpha = \frac{2}{\sqrt{5}}$$

$$\alpha = 63.4^\circ$$

For $R \sin(x + \alpha)$ the maximum, R , is achieved when $x + \alpha = \dots 90, 450 \dots$

For $R \cos(x + \alpha)$ the maximum, R , is achieved when $x + \alpha = \dots 0, 360 \dots$

$$\sin^2 A = 1 - \cos^2 A$$

Using recurrence relations to find approximate roots



$f(x) = \dots$ is continuous and changes sign between $x = a$ and $x = b$, therefore $f(x)$ has a root between $x = a$ and $x = b$. This proves that the root is $x = \dots$ to 2* dp

$x^2 - 5x = 3$ has a root in the interval $5 < x < 6$.

Show that the root of this equation also solves the equation $x = \sqrt{5x + 3}$

$$\begin{aligned} x^2 - 5x &= 3 \\ \therefore x^2 &= 5x + 3 \\ \therefore x &= \sqrt{5x + 3} \end{aligned}$$

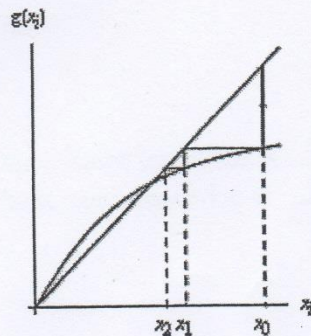
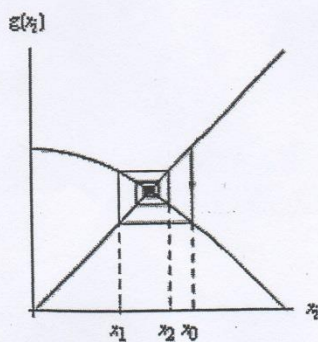
Take $x_0 = 5$ and use iteration to find 4 improved approximations to the root of the equation $x^2 - 5x = 3$

$$\begin{aligned} x_0 &= 5 & x_1 &= \sqrt{5x_0 + 3} = \sqrt{28} = 5.292 \\ x_2 &= \sqrt{5x_1 + 3} = \sqrt{29.4575} = 5.427 \\ x_3 &= \sqrt{5x_2 + 3} = \sqrt{30.1374} = 5.490 \\ x_4 &= \sqrt{5x_3 + 3} = \sqrt{30.4488} = 5.518 \end{aligned}$$

Prove that the root is 5.5 to 1 d.p.

$$\begin{aligned} \text{Let } f(x) &= x^2 - 5x - 3 \\ f(5.55) &= 0.0525 \\ f(5.45) &= -0.5475 \end{aligned}$$

Because there is a change of sign and because $f(x)$ is continuous, a root of the equation is 5.5



use this to get to the curve

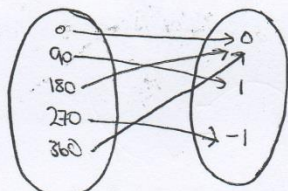
use this to get to $y = x$



Range & Domain, Inverse, Composites

Domain and Range

$$f(x) = a \cdot x$$

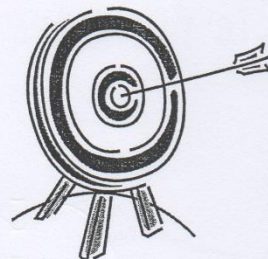


DOMAIN

RANGE

Domain is the starting numbers

Range is the ending numbers



This is an example of a many-one function.

Inverse

$$f(x) = 5x - 2$$

$$f^{-1}(x) = \frac{x+2}{5}$$

going from the range numbers to the domain numbers

Composite

$$f(x) = x^2$$

$$g(x) = 2x + 3$$

$$fg(x) = \text{do } g \text{ first} = (2x+3)^2$$

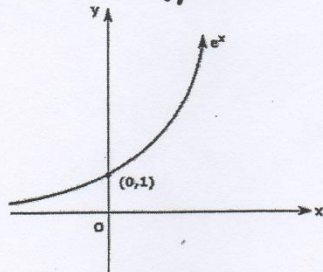
$$gf(x) = \text{do } f \text{ first} = 2x^2 + 3$$

~~def~~

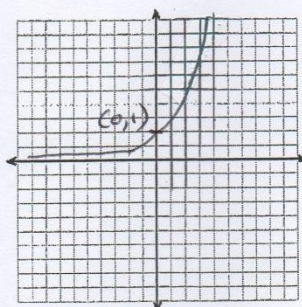
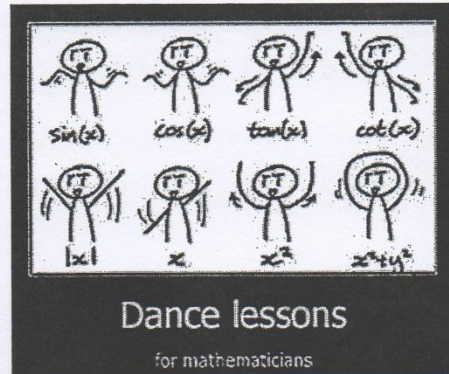
function

A function is a mapping such that every element of the domain is mapped to exactly one element of the range.

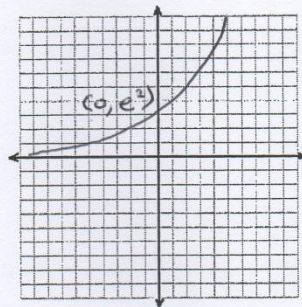
Graph sketching is fun (yes it is)



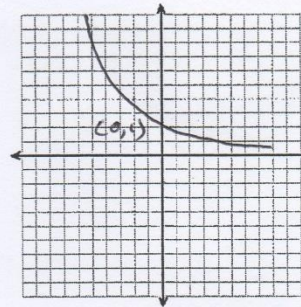
$$f(x) = e^x$$



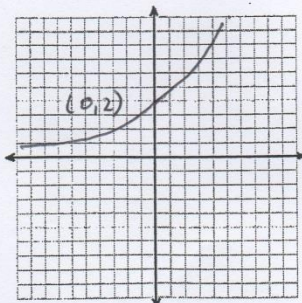
$$f(2x) \uparrow = e^{2x}$$



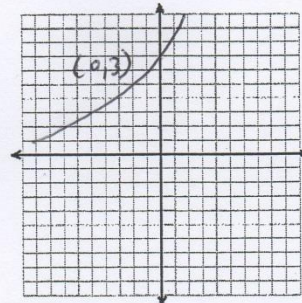
$$f(x+2) \uparrow = e^{x+2}$$



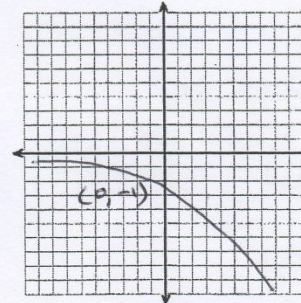
$$f(-x) \uparrow = e^{-x}$$



$$2f(x) \uparrow = 2e^x$$



$$f(x)+2 \uparrow = e^x + 2$$



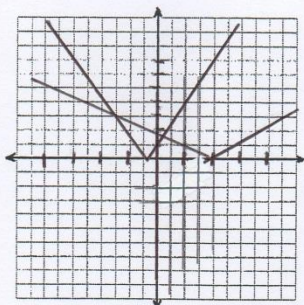
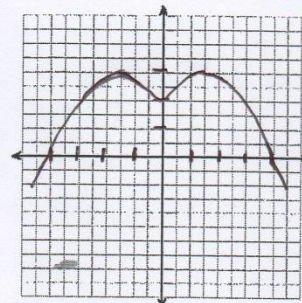
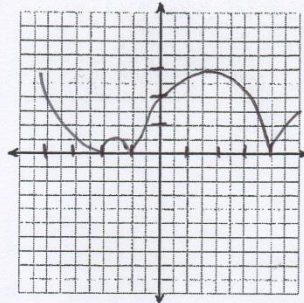
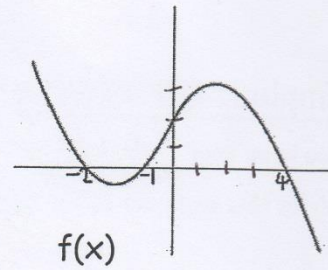
$$-f(x) \uparrow = -e^x$$

Remember to show the x-intercept: and the y-intercept:

Now you can apply combinations of these transformations to e^x and $\ln x$
Always label the equation of the asymptote!



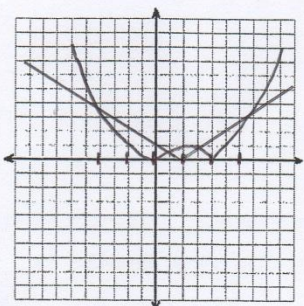
Modulus Equations need to be solved using a graph so you know which equations to put together!



$$|3x + 1| = |x - 2|$$

top tips...

Exaggerate the steepness of the steeper line to make sure you get all the intercepts



$$|x(x-2)| = |x-1|$$