# C4 Revision Questions

## 1 Algebra and functions

**What students need to learn:**

| | |
|---|---|
| Rational functions. Partial fractions (denominators not more complicated than repeated linear terms). | Partial fractions to include denominators such as $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx + d)^2$.<br><br>The degree of the numerator may equal or exceed the degree of the denominator. Applications to integration, differentiation and series expansions.<br><br>Quadratic factors in the denominator such as $(x^2 + a)$, $a > 0$, are *not* required. |
| Simple cases of integration using partial fractions. | Integration of rational expressions such as those arising from partial fractions, eg $\frac{2}{3x+5}$, $\frac{3}{(x-1)^2}$.<br><br>Note that the integration of other rational expressions, such as $\frac{x}{x^2+5}$ and $\frac{2}{(2x-1)^4}$ is also required (see above paragraphs). |

**1.** $f(x) = \dfrac{27x^2 + 32x + 16}{(3x+2)^2(1-x)}$, $|x| < \dfrac{2}{3}$.

Given that f(*x*) can be expressed in the form $f(x) = \dfrac{A}{(3x+2)} + \dfrac{B}{(3x+2)^2} + \dfrac{C}{(1-x)}$,

(*a*) find the values of *B* and *C* and show that $A = 0$. **(4)**

(*b*) Hence, or otherwise, find the series expansion of f(*x*), in ascending powers of *x*, up to and including the term in $x^2$. Simplify each term. **(6)**

(*c*) Find the percentage error made in using the series expansion in part (*b*) to estimate the value of f(0.2). Give your answer to 2 significant figures. **(4)**

**2.** $f(x) = \dfrac{3x^2 + 16}{(1-3x)(2+x)^2} = \dfrac{A}{(1-3x)} + \dfrac{B}{(2+x)} + \dfrac{C}{(2+x)^2}$, $|x| < \frac{1}{3}$.

(*a*) Find the values of *A* and *C* and show that $B = 0$. **(4)**

(*b*) Hence, or otherwise, find the series expansion of f(*x*), in ascending powers of *x*, up to and including the term in $x^3$. Simplify each term. **(7)**

**3.** ($a$) Express $\dfrac{2x-1}{(x-1)(2x-3)}$ in partial fractions. **(3)**

($b$) Given that $x \geq 2$, find the general solution of the differential equation

$$(2x-3)(x-1)\frac{\mathrm{d}y}{\mathrm{d}x} = (2x-1)y.$$ **(5)**

($c$) Hence find the particular solution of this differential equation that satisfies $y = 10$ at $x = 2$, giving your answer in the form $y = \mathrm{f}(x)$. **(4)**

**4.**

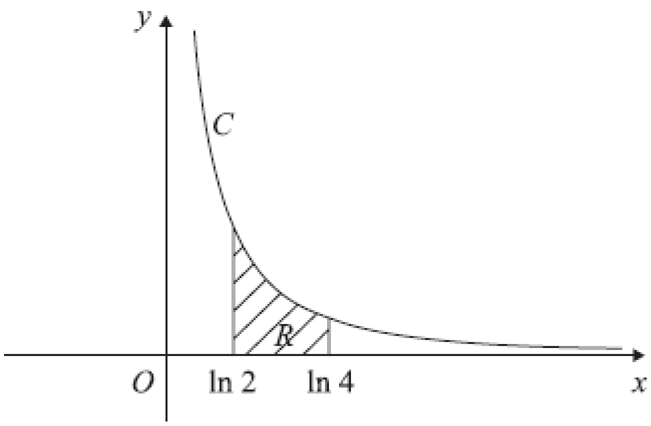



***Figure 3***

The curve $C$ has parametric equations $x = \ln(t+2)$, $y = \dfrac{1}{(t+1)}$, $t > -1$.

The finite region $R$ between the curve $C$ and the $x$-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

($a$) Show that the area of $R$ is given by the integral $\displaystyle\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t$. **(4)**

($b$) Hence find an exact value for this area. **(6)**

($c$) Find a Cartesian equation of the curve $C$, in the form $y = \mathrm{f}(x)$. **(4)**

($d$) State the domain of values for $x$ for this curve. **(1)**

**2 Coordinate geometry in the (*x*, *y*) plane**

**What students need to learn:**

| | |
|---|---|
| Parametric equations of curves and conversion between Cartesian and parametric forms. | Students should be able to find the area under a curve given its parametric equations. Students will *not* be expected to sketch a curve from its parametric equations. |

**5.**

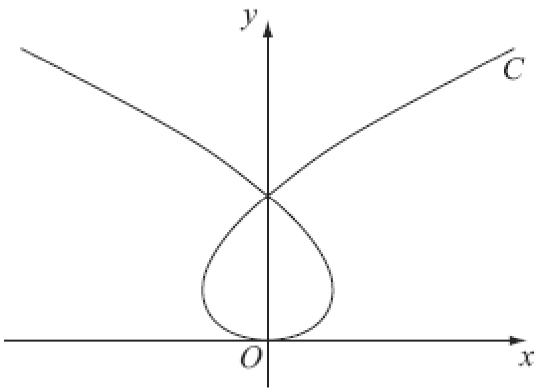



***Figure 3***

The curve $C$ shown in Figure 3 has parametric equations $x = t^3 - 8t$, $y = t^2$ where $t$ is a parameter. Given that the point $A$ has parameter $t = -1$,

(*a*) find the coordinates of $A$. **(1)**

The line $l$ is the tangent to $C$ at $A$.

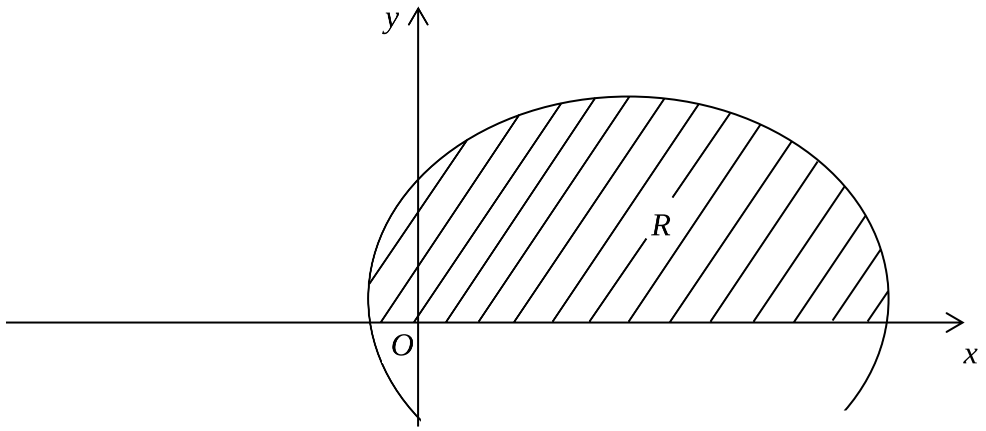
(*b*) Show that an equation for $l$ is $2x - 5y - 9 = 0$. **(5)**

The line $l$ also intersects the curve at the point $B$.

(*c*) Find the coordinates of $B$. **(6)**

**6**.

***Figure 2***




The curve shown in Figure 2 has parametric equations $t - 2\sin t$,   $y = 1 - 2\cos t, 0 \le t \le 2\pi$.

(*a*) Show that the curve crosses the *x*-axis where $t = \dfrac{\pi}{3}$ and $t = \dfrac{5\pi}{3}$. **(2)**

The finite region *R* is enclosed by the curve and the *x*-axis, as shown shaded in Figure 2.

(*b*) Show that the area *R* is given by the integral $\displaystyle\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 \, \mathrm{d}t$. **(3)**

(*c*) Use this integral to find the exact value of the shaded area. **(7)**

### 3 Sequences and series

**What students need to learn:**

Binomial series for any rational $n$.

For $|x| < \frac{b}{a}$, students should be able to obtain the expansion of $(ax + b)^n$, and the expansion of rational functions by decomposition into partial fractions.

**7.** (*a*) Use the binomial theorem to expand $(8 - 3x)^{\frac{1}{3}}$, $|x| < \frac{8}{3}$, in ascending powers of $x$, up to and including the term in $x^3$, giving each term as a simplified fraction. **(5)**

(*b*) Use your expansion, with a suitable value of $x$, to obtain an approximation to $\sqrt[3]{7.7}$. Give your answer to 7 decimal places. **(2)**

**8.** $f(x) = (2 - 5x)^{-2}$, $|x| < \frac{2}{5}$.

Find the binomial expansion of $f(x)$, in ascending powers of $x$, as far as the term in $x^3$, giving each coefficient as a simplified fraction. **(5)**

**4 Differentiation**

**What students need to learn:**

| | |
|---|---|
| Differentiation of simple functions defined implicitly or parametrically. | The finding of equations of tangents and normals to curves given parametrically or implicitly is required. |
| Exponential growth and decay. | Knowledge and use of the result $\frac{d}{dx}(a^x) = a^x \ln a$ is expected. |
| Formation of simple differential equations. | Questions involving connected rates of change may be set. |
| Analytical solution of simple first order differential equations with separable variables. | General and particular solutions will be required. |

**9.** Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 $cm^3s^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 $cm^2$.

(*a*) Show that at time *t* seconds, the height *h* cm of liquid in the cylinder satisfies the differential equation $\frac{dh}{dt} = 0.4 - k\sqrt{h}$, where *k* is a positive constant. **(3)**

When $h = 25$, water is leaking out of the hole at 400 $cm^3s^{-1}$.

(*b*) Show that $k = 0.02$. **(1)**

(*c*) Separate the variables of the differential equation $\frac{dh}{dt} = 0.4 - 0.02\sqrt{h}$ to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by $\int_0^{100} \frac{50}{20-\sqrt{h}}\,dh$. **(2)**

Using the substitution $h = (20 - x)^2$, or otherwise,

(*d*) find the exact value of $\int_0^{100} \frac{50}{20-\sqrt{h}}\,dh$. **(6)**

(*e*) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second. **(1)**

**10**. A curve is described by the equation $x^3 - 4y^2 = 12xy$.

(*a*) Find the coordinates of the two points on the curve where $x = -8$. **(3)**

(*b*) Find the gradient of the curve at each of these points. **(6)**

**11**.*(a )*Given that $y = 2^x$, and using the result $2^x = e^{x \ln 2}$, or otherwise, show that $\frac{dy}{dx} = 2^x \ln 2$.

**(2)**

(*b*) Find the gradient of the curve with equation $y = 2^{(x^2)}$ at the point with coordinates (2, 16).

**(4)**

**12**. A curve has parametric equations

$$x = 7\cos t - \cos 7t, \quad y = 7\sin t - \sin 7t, \quad \frac{\pi}{8} < t < \frac{\pi}{3}$$

(*a*) Find an expression for $\frac{dy}{dx}$ in terms of *t*. You need not simplify your answer.

**(3)**

(*b*) Find an equation of the normal to the curve at the point where $t = \frac{\pi}{6}$. Give your answer in its simplest exact form.

**(6)**

**13**.A set of curves is given by the equation $\sin x + \cos y = 0.5$.

(*a*) Use implicit differentiation to find an expression for $\frac{dy}{dx}$. **(2)**

For $-\pi < x < \pi$ and $-\pi < y < \pi$,

(*b*) find the coordinates of the points where $\frac{dy}{dx} = 0$. **(5)**

**14**. A curve *C* has the equation $y^2 - 3y = x^3 + 8$.

(*a*) Find $\frac{dy}{dx}$ in terms of *x* and *y*. **(4)**

(*b*) Hence find the gradient of *C* at the point where $y = 3$. **(3)**

**15**. A curve *C* is described by the equation $3x^2 + 4y^2 - 2x + 6xy - 5 = 0$.

Find an equation of the tangent to *C* at the point (1, –2), giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

**(7)**

**16.**

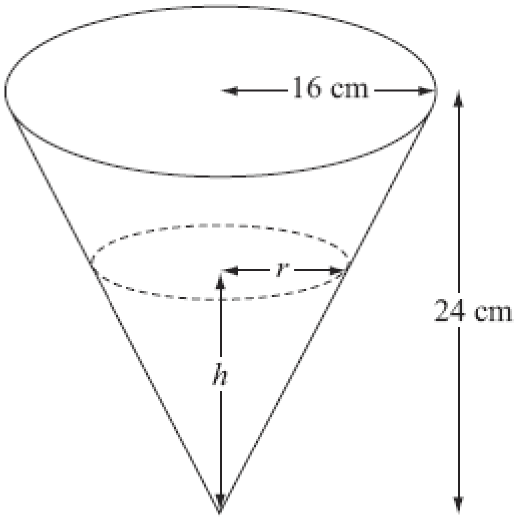



***Figure 2***

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is $h$ cm, the surface of the water has radius $r$ cm and the volume of water is $V$ cm$^3$.

($a$) Show that $V = \dfrac{4\pi h^3}{27}$. **(2)**

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \dfrac{1}{3}\pi r^2 h$ .]

Water flows into the container at a rate of 8 cm$^3$ s$^{-1}$.

($b$) Find, in terms of $\pi$, the rate of change of $h$ when $h = 12$. **(5)**

**17.** The volume of a spherical balloon of radius $r$ cm is $V$ cm$^3$, where $V = \frac{4}{3}\pi r^3$.

($a$) Find $\dfrac{dV}{dr}$. **(1)**

The volume of the balloon increases with time $t$ seconds according to the formula

$$\frac{dV}{dt} = \frac{1000}{(2t+1)^2}, \quad t \geq 0.$$

($b$) Using the chain rule, or otherwise, find an expression in terms of $r$ and $t$ for $\dfrac{dr}{dt}$. **(2)**

($c$) Given that $V = 0$ when $t = 0$, solve the differential equation $\dfrac{dV}{dt} = \dfrac{1000}{(2t+1)^2}$, to obtain $V$ in terms of $t$. **(4)**

($d$) Hence, at time $t = 5$,

(i) find the radius of the balloon, giving your answer to 3 significant figures, **(3)**

(ii) show that the rate of increase of the radius of the balloon is approximately $2.90 \times 10^{-2}$ cm s$^{-1}$. **(2)**

| | |
|---|---|
| Integration of $e^x$, $\frac{1}{x}$, $\sin x$, $\cos x$. | To include integration of standard functions such as $\sin 3x$, $\sec^2 2x$, $\tan x$, $e^{5x}$, $\frac{1}{2x}$.<br><br>Students should recognise integrals of the form $\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + c$.<br><br>Students are expected to be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$. |
| Simple cases of integration by substitution and integration by parts. These methods as the reverse processes of the chain and product rules respectively. | Except in the simplest of cases the substitution will be given.<br><br>The integral $\int \ln x \, dx$ is required.<br><br>More than one application of integration by parts may be required, for example $\int x^2 e^x \, dx$. |

**18.** (*a*) Find $\int \tan^2 x \; dx$. **(2)**

(*b*) Use integration by parts to find $\int \frac{1}{x^3} \ln x \, dx$. **(4)**

(*c*) Use the substitution $u = 1 + e^x$ to show that

$$\int \frac{e^{3x}}{1+e^x}\,dx = \frac{1}{2}e^{2x} - e^x + \ln(1 + e^x) + k,$$

where $k$ is a constant. **(7)**

**19**. Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{(2x-1)}}\,dx.$$

**(8)**

**20**. (i) Find $\int \ln\left(\frac{x}{2}\right) dx$. **(4)**

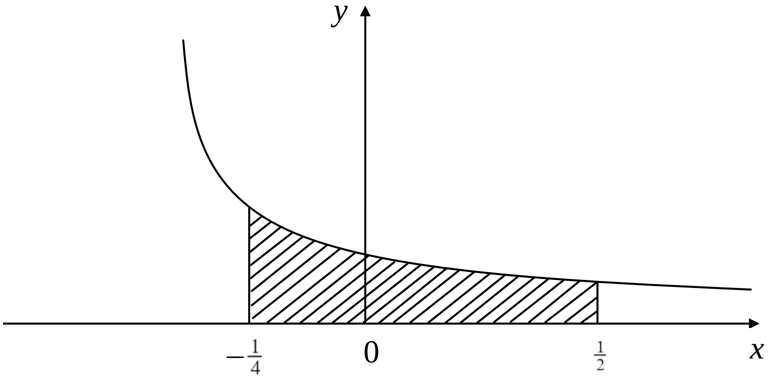
(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \; dx$. **(5)**

Evaluation of volume of revolution.

$\pi \int y^2 \, dx$ is required, but *not* $\pi \int x^2 \, dy$. Students should be able to find a volume of revolution, given parametric equations.

**21.**

***Figure 1***




The curve with equation $y = \dfrac{1}{3(1+2x)}$, $x > -\frac{1}{2}$, is shown in Figure 1.

The region bounded by the lines $x = -\frac{1}{4}$, $x = \frac{1}{2}$, the $x$-axis and the curve is shown shaded in Figure 1. This region is rotated through 360 degrees about the $x$-axis.

(*a*) Use calculus to find the exact value of the volume of the solid generated. **(5)**

***Figure 2***

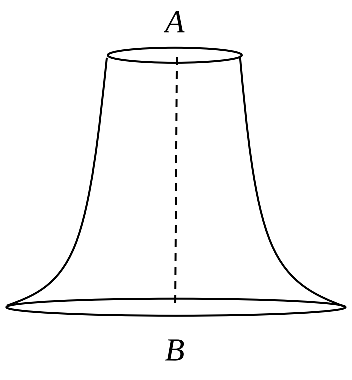



Figure 2 shows a paperweight with axis of symmetry *AB* where *AB* = 3 cm. *A* is a point on the top surface of the paperweight, and *B* is a point on the base of the paperweight. The paperweight is geometrically similar to the solid in part (*a*).

(*b*) Find the volume of this paperweight. **(2)**

**22.** ***Figure 1***

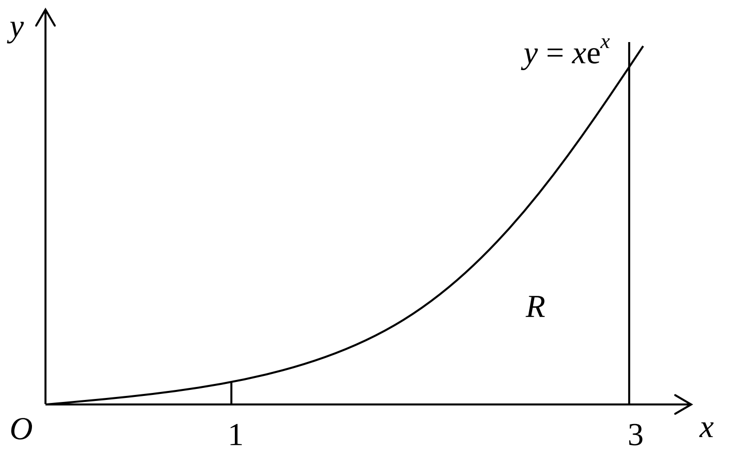



Figure 1 shows the finite region $R$, which is bounded by the curve $y = xe^x$, the line $x = 1$, the line $x = 3$ and the $x$-axis. The region $R$ is rotated through 360 degrees about the $x$-axis.
Use integration by parts to find an exact value for the volume of the solid generated. **(8)**

**23.**

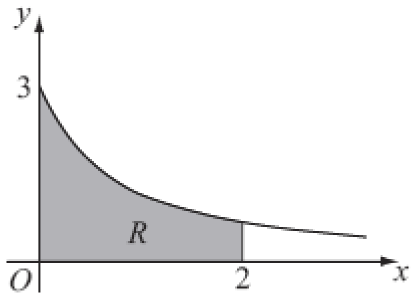



***Figure 1***

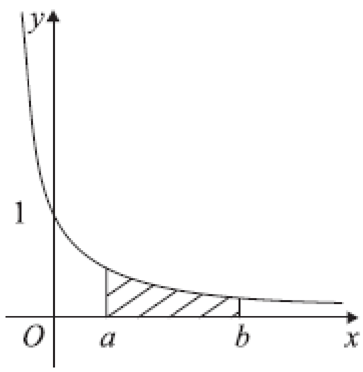
Figure 1 shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$. The region $R$ is bounded by the curve, the $x$-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(*a*) Use integration to find the area of $R$. **(4)**

The region $R$ is rotated 360° about the $x$-axis.

(*b*) Use integration to find the exact value of the volume of the solid formed. **(5)**

**24.**




The curve shown in Figure 2 has equation $y = \dfrac{1}{(2x+1)}$.

The finite region bounded by the curve, the $x$-axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the $x$-axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of $a$ and $b$.

**(5)**

Numerical integration of functions.

Application of the trapezium rule to functions covered in C3 and C4. Use of increasing number of trapezia to improve accuracy and estimate error will be required. Questions will not require more than three iterations.

Simpson's Rule is *not* required.

**25.**

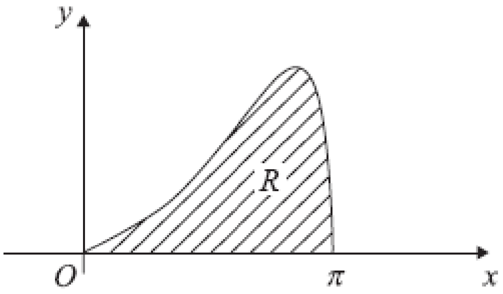



***Figure 1***

The curve shown in Figure 1 has equation $e^x\sqrt{(\sin x)}$, $0 \le x \le \pi$. The finite region $R$ bounded by the curve and the $x$-axis is shown shaded in Figure 1.

(*a*) Copy and complete the table below with the values of $y$ corresponding to $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$, giving your answers to 5 decimal places. **(2)**

| $x$ | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ |
|---|---|---|---|---|---|
| $y$ | 0 | | | 8.87207 | 0 |

(*b*) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region $R$. Give your answer to 4 decimal places. **(4)**

**26.** (*a*) Given that $y = \sec x$, complete the table with the values of $y$ corresponding to $x = \frac{\pi}{16}, \frac{\pi}{8}$ and $\frac{\pi}{4}$. **(2)**

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | $\frac{\pi}{16}$ | $\frac{\pi}{8}$ | $\frac{3\pi}{16}$ | $\frac{\pi}{4}$ |
| $y$ | 1 | | | 1.20269 | |

(*b*) Use the trapezium rule, with all the values for $y$ in the completed table, to obtain an estimate for $\int_0^{\frac{\pi}{4}} \sec x \;\; \mathrm{d}x$. Show all the steps of your working and give your answer to 4 decimal places. **(3)**

The exact value of $\int_0^{\frac{\pi}{4}} \sec x \;\; \mathrm{d}x$ is $\ln(1 + \sqrt{2})$.

(*c*) Calculate the % error in using the estimate you obtained in part (*b*). **(2)**

**27**. $I = \int_0^5 \mathrm{e}^{\sqrt{(3x+1)}} \;\; \mathrm{d}x$.

(*a*) Given that $y = \mathrm{e}^{\sqrt{(3x + 1)}}$, copy and complete the table with the values of $y$ corresponding to $x = 2$, 3 and 4. **(2)**

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
| $y$ | $\mathrm{e}^1$ | $\mathrm{e}^2$ | | | | $\mathrm{e}^4$ |

(*b*) Use the trapezium rule, with all the values of $y$ in the completed table, to obtain an estimate for the original integral $I$, giving your answer to 4 significant figures. **(3)**

(*c*) Use the substitution $t = \sqrt{(3x + 1)}$ to show that $I$ may be expressed as $\int_a^b kt\mathrm{e}^t \;\; \mathrm{d}t$, giving the values of $a$, $b$ and $k$. **(5)**

(*d*) Use integration by parts to evaluate this integral, and hence find the value of $I$ correct to 4 significant figures, showing all the steps in your working. **(5)**

## 6 Vectors

### What students need to learn:

| | |
|---|---|
| Vectors in two and three dimensions. | |
| Magnitude of a vector. | Students should be able to find a unit vector in the direction of **a**, and be familiar with $\lvert a \rvert$. |
| Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations. | |
| Position vectors. | $\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$. |
| The distance between two points. | The distance $d$ between two points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is given by $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$. |
| Vector equations of lines. | To include the forms $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + t(\mathbf{d} - \mathbf{c})$. <br> Intersection, or otherwise, of two lines. |
| The scalar product. Its use for calculating the angle between two lines. | Students should know that for $\overrightarrow{OA} = \mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\overrightarrow{OB} = \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ and $\cos \angle AOB = \dfrac{\mathbf{a} \cdot \mathbf{b}}{\lvert\mathbf{a}\rvert \lvert\mathbf{b}\rvert}$. <br> Students should know that if $\mathbf{a} \cdot \mathbf{b} = 0$, and that **a** and **b** are non-zero vectors, then **a** and **b** are perpendicular. |

**28.** With respect to a fixed origin $O$ the lines $l_1$ and $l_2$ are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where $\lambda$ and $\mu$ are parameters and $p$ and $q$ are constants. Given that $l_1$ and $l_2$ are perpendicular,

(*a*) show that $q = -3$. **(2)**

Given further that $l_1$ and $l_2$ intersect, find

(*b*) the value of $p$, **(6)**

(*c*) the coordinates of the point of intersection. **(2)**

The point $A$ lies on $l_1$ and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point $C$ lies on $l_2$.

Given that a circle, with centre $C$, cuts the line $l_1$ at the points $A$ and $B$,

(*d*) find the position vector of $B$. **(3)**

**29**. The line $l_1$ has vector equation $\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k})$, where $\lambda$ is a parameter. The point $A$ has coordinates $(4, 8, a)$, where $a$ is a constant. The point $B$ has coordinates $(b, 13, 13)$, where $b$ is a constant. Points $A$ and $B$ lie on the line $l_1$.

(*a*) Find the values of $a$ and $b$. **(3)**

Given that the point $O$ is the origin, and that the point $P$ lies on $l_1$ such that $OP$ is perpendicular to $l_1$,

(*b*) find the coordinates of $P$. **(5)**

(c) Hence find the distance $OP$, giving your answer as a simplified surd. **(2)**

**30**. The points $A$ and $B$ have position vectors $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ respectively. The line $l_1$ passes through the points $A$ and $B$.

(*a*) Find the vector $\overrightarrow{AB}$. **(2)**

(*b*) Find a vector equation for the line $l_1$. **(2)**

A second line $l_2$ passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$. The line $l_1$ meets the line $l_2$ at the point $C$.

(*c*) Find the acute angle between $l_1$ and $l_2$. **(3)**

(*d*) Find the position vector of the point $C$. **(4)**

**31** The point $A$ has position vector $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and the point $B$ has position vector $\mathbf{b} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$, relative to an origin $O$.

(*a*) Find the position vector of the point $C$, with position vector $\mathbf{c}$, given by $\mathbf{c} = \mathbf{a} + \mathbf{b}$. **(1)**

(*b*) Show that $OACB$ is a rectangle, and find its exact area. **(6)**

The diagonals of the rectangle, $AB$ and $OC$, meet at the point $D$.

(*c*) Write down the position vector of the point $D$. **(1)**

(*d*) Find the size of the angle $ADC$. **(6)**

Answers

1) a) B=4, C=3 b) $4 + \frac{39}{4}x^2$ $c)$ 1.1 %
2) a) A=3, C = 4 b) $4 + 8x + \frac{111}{4}x^2 + \frac{161}{2}x^3 + \ldots$
3) a) $\frac{4}{2x-3} - \frac{1}{x-1}$ b) ln|y| = 2ln|2x - 3| - ln|x - 1| + ln A c) $y = \frac{10(2x-3)^2}{x-1}$
4) a) ln ½ b) $y = \frac{1}{e^x-1}$ c) x > 0
5) a) (7,1) c) $(\frac{441}{8}, \frac{81}{4})$
6) c) $4\pi + 3\sqrt{3}$
7) a) $2 - \frac{1}{4}x - \frac{1}{32}x^2 - \frac{5}{768}x^3 - \cdots$ b) 1.9746810
8) $\frac{1}{4} + \frac{5}{4}x + \frac{75}{16}x^2 + \frac{125}{8}x^3 + \cdots$
9) $d)\ 2000 \ln 2 - 1000$ e) 6 minutes 26 seconds
10) $a)(-8,16) and\ (-8,8)$ b) -3,0
11) $b)\ 64\ ln2$
12) a) $\frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t}$ b) $-\sqrt{3}$ c) $y = \frac{1}{\sqrt{3}}x$
13) a) $\frac{dy}{dx} = \frac{\cos x}{\sin y}$ b) $(\frac{\pi}{2}, \frac{2\pi}{3})$ $(\frac{\pi}{2}, \frac{-2\pi}{3})$
14) a) $\frac{3x^2}{2y-3}$ b) 4
15) 4x + 5y + 6 =0
16) b) $\frac{1}{8\pi}$
17) a) $4\pi r^2$ b) $\frac{250}{\pi r^2(2t+1)^2}$ c) $-\frac{500}{2t+1} + 500$ d) i) 4.77
18) a) tan x – x + c b) $-\frac{1}{2x^2}\ln x + \frac{1}{2}(-\frac{1}{2x^2}) + c$
19) 16
20) i) $x \ln\frac{x}{2} - x + c$ ii) $\frac{\pi}{8} + \frac{1}{4}$
21) a) $\frac{\pi}{12}$ b) $\frac{16\pi}{3}$ $\text{cm}^3$
22) $\frac{13}{4}\pi e^6 - \frac{\pi e^2}{4}$
23) a) 3 b) $\frac{9}{4}\pi \ln 9$
24) $\frac{\pi(b-a)}{(2a+1)(2b+1)}$
25) a) 1.84432, 4.81048, 4.81047 b) 12.1948
26) a) 1.01959, 1.08239, 1.20269, 1.41421 b) 0.8859 c) 0.514%
27) a) $e^{\sqrt{7}}$, $e^{\sqrt{10}}$, $e^{\sqrt{13}}$ b) 110.6 d) 109.2
28) b) p = 1 c) $\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ d) $\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$
29) a) a = 18, b = 9 b) P(6, 10, 16) c) $14\sqrt{2}$
30) a) $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ b) $\mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ c) $\theta = \frac{\pi}{4}$
31) a) $\mathbf{c} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ b) $9\sqrt{2}$ c) $\frac{3}{2}\boldsymbol{i} + \frac{3}{2}\boldsymbol{j} - \frac{3}{2}\boldsymbol{k}$ d) 109.5°