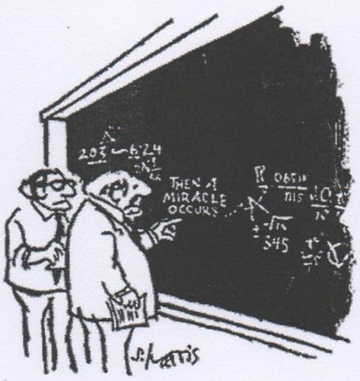
BHASVIC
mαths

C3 C4 S2 Survival Kit

Core 3

Core 4



"I think you should be more explicit here in step two."

Statistics 2

Name ....

Implicit Differentiation

You know that $\frac{d}{dx}(3x+2)^6 = 6(3x+2)^5 \times \frac{d}{dx}(3x+2) =$

Write this again but put 'y' instead of '3x+2'

$y^4 = 3\sin 2x$

$4y^3 \frac{dy}{dx} = 6\cos 2x$

$\frac{dy}{dx} = \frac{6\cos 2x}{4y^3}$

<u>Key example</u>

$y = a^x$

$\ln y = \ln a^x$

$\ln y = x \ln a$

Differentiate both sides then rearrange to get $\frac{dy}{dx}$

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x\ln a)$

$\frac{dy}{dx}\frac{1}{y} = \ln a$.

$\frac{dy}{dx} = \ln(a)y$

$\frac{dy}{dx} = a^x \ln(a)$

Partial Fractions (an example)

$$\frac{2x}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}$$

$$2x = A(x-1)(2x+1) + B(2x+1) + C(x-1)(x-1)$$

To get simultaneous equations sub in different values of x

let $x = 1$

$2 = 3B$

$\frac{2}{3} = B$

let $x = -\frac{1}{2}$

$-1 = -4C$

$C = \frac{1}{4}$

let $x = 0$

$0 = -A + B + C$

$0 = -A + \frac{2}{3} + \frac{1}{4}$

$A = \frac{11}{12}$

NOTE: $\frac{3x-x^2+8}{(x+1)(3-x)} = 1 + \frac{1}{(x+1)} + \frac{2}{(3-x)}$ (Divide first)

No offence but you might have gone wrong... substitute a new value of x into the original and into your answer to check they are the same!

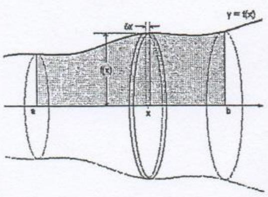
You HAVE to be able to integrate!

It helps to start by getting numbers outside the integral so they don't confuse things

$$\int \frac{-1}{3(x-2)}\,dx = \frac{-1}{3}\int \frac{1}{x-2}\,dx = -\frac{1}{3}\ln|x-2| + c$$

Good news! You only have to integrate the 9 derivatives.
Estimate what the thing you're integrating came from then
Differentiate this to see how to Adjust it into the solution.

$\cos(ax+b)$	has come from $\frac{1}{a}\sin(ax+b)$
$\sin(ax+b)$	has come from $-\frac{1}{a}\cos(ax+b)$
$\sec^2(ax+b)$	has come from $\frac{1}{a}\tan(ax+b)$
$\sec(ax+b)\tan(ax+b)$	has come from $\frac{1}{a}\sec(ax+b)$
$\operatorname{cosec}(ax+b)\cot(ax+b)$	has come from $-\frac{1}{a}\operatorname{cosec}(ax+b)$
$\operatorname{cosec}^2(ax+b)$	has come from $-\frac{1}{a}\cot(ax+b)$
e^{ax+b}	has come from $\frac{1}{a}e^{ax+b}$
$\frac{1}{ax+b}$	has come from $\frac{1}{a}\ln|ax+b|$
$(ax+b)^n$	has come from $\frac{1}{a(n+1)}(ax+b)^{n+1}$

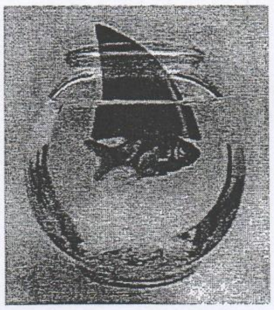


Volumes of Revolution: use limits given in the question.

$$V = \int \pi y^2\,dx$$

Integrating things which are not the 9 derivatives:

The 9 can be disguised to look more complicated than they really are:



$$\int \frac{1}{\cos^2 x}\,dx = \int \sec^2 x\,dx = \tan x + c$$

$$\int \frac{1}{x^2-4x+4}\,dx = \int \frac{1}{(x-2)^2}\,dx = -\frac{1}{x-2} + c$$

$$\int \frac{1}{\sec x \operatorname{cosec} x}\,dx = \int \sin x \cos x\,dx$$

$$= \int \frac{1}{2} \sin 2x\,dx$$

$$= -\frac{1}{4}\cos 2x + c$$

There are some things you should simply know how to integrate:

$$\int \sin^2 x\,dx = \int \frac{1}{2}(1-\cos 2x)\,dx$$

$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + c$$

$$\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx$$

$$= \tan x - x + c$$

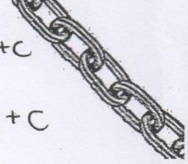
$$\int \cos^2 x\,dx = \int \frac{1}{2}(1+\cos 2x)\,dx$$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + c$$

$$\int \cot^2 x\,dx = \int (\operatorname{cosec}^2 x - 1)\,dx$$

$$= -\cot x - x + c$$

Sometimes one of the 9 is applied to something more complicated than (ax+b) but the derivative is there too because of the chain rule:



$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$

$\int f'(x)[f(x)]^n\,dx = \frac{1}{n+1}[f(x)]^{n+1} + c$

$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$\int f'(x)\cos[f(x)]\,dx = \sin[f(x)] + c$

Fractions, if not the chain rule example above, can be simplified either with partial fractions (if the bottom can be factorised) or algebraic division (if the top is equal to or heavier than the bottom)

$\int \frac{x^2+3}{(x-1)(x+3)}\,dx =$

For this one use:

algebraic division

$\int \frac{2x}{(x-1)(x+3)}\,dx =$

For this one use:

partial fractions

Integration by Parts

(the integral version of the chain rule)

$$\int (u)(dv)\,dx = (u)(v) - \int (v)\left(\frac{du}{dx}\right)dx$$

$$\int x\sin x\,dx = -x\cos x - \int -\cos x\,dx$$
$$= -x\cos x + \int \cos x\,dx$$
$$= -x\cos x + \sin x + c$$

$u = x$

$\frac{du}{dx} = 1$

$\frac{dv}{dx} = \sin x$

$v = -\cos x$

$$\int \ln x\,dx = x\ln x - \int \frac{x}{x}\,dx$$
$$= x\ln x - \int 1\,dx$$
$$= x\ln x - x + c$$

$u = 1$ $\quad \frac{dv}{dx} = \ln x$

$\frac{du}{dx} = 0$ $\quad v = ?$

$u = \ln x$ $\quad \frac{dv}{dx} = 1$

$\frac{du}{dx} = \frac{1}{x}$ $\quad v = x$

THE RULES

HOW TO LIVE YOUR LIFE

Integration by Substitution (you will be told what substitution to use)

- Limits? Change them from x to u before you forget
- Differentiate The Substitution to get dx =
- Sub In Obvious Things e.g. dx = and anything really obvious. Usually some things will cancel at this stage ☺
- If there any x left, write them in terms of u then integrate!
- NO LIMITS? Don't forget to change everything back to x

$\int 5x(7x^2+2)^3 dx$

$= \int 5xu^3 \cdot \frac{du}{14x}$

$= \frac{5}{14}\int u^3 du$

$= \frac{5}{14} \times \frac{1}{4}u^4 + c$

$= \frac{5}{56}u^4 + c$

$= \frac{5}{56}(7x^2+2)^4 + c$

$u = 7x^2+2$

$\frac{du}{dx} = 14x$

replace dx by $\frac{du}{14x}$

$= \int_0^1 4x\sqrt{5-3x}\, dx$

$= \int_5^2 \frac{4xu^{\frac{1}{2}}}{-3} du$

$= \frac{-4}{3}\int_5^2 xu^{\frac{1}{2}} du$

$= -\frac{4}{3}\int_5^2 \frac{(5-u)}{3}u^{\frac{1}{2}} du$

$= \frac{-4}{9}\int_5^2 (5-u)u^{\frac{1}{2}} du$

$= \frac{-4}{9}\int_5^2 5u^{\frac{1}{2}} - u^{\frac{3}{2}} du$

$= \frac{-4}{9}\left[\frac{10}{3}u^{\frac{3}{2}} - \frac{2}{5}u^{\frac{5}{2}}\right]_5^2$

$= -3.184599 + 6.625386$

$= 3.44$ (2dp)

$u = 5-3x$

$x = \frac{5-u}{3}$

$\frac{du}{dx} = -3$

replace $dx = \frac{du}{-3}$

$x=0 \rightarrow u=5$

$x=1 \rightarrow u=2$

Trapezium Rule (this is the same as C2 but with C4 equations)

x	x_1	x_2	…	…	…	x_n	
f(x)	y_1					y_n	$\frac{h}{2}[y_1 + y_n + 2(\text{other y values})]$

e.g. $\int_0^1 (x^2+1)^3 \, dx$ with 5 ordinates to 3 d.p.

x	0	0.25	0.5	0.75	1
$f(x)$	1	1.1994	1.9531	3.8147	8

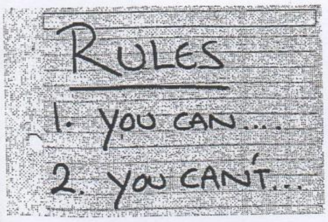
$$\therefore \int_0^1 (x^2+1)^3 \, dx \approx \frac{0.25}{2}\left[1 + 8 + 2(1.1944 + 1.9531 + 3.8147)\right]$$

$$= 0.125\left[9 + 2 \times 6.9622\right]$$

$$= 0.125 \times 22.9244$$

$$= 2.86555$$

$$= 2.866 \text{ (3 d.p.)}$$



Binomial Expansion

It has to be $(1+x)^n$!!

$$(2+3x)^n = \left[2\left(1+\frac{3}{2}x\right)\right]^n = 2^n\left(1+\frac{3}{2}x\right)^n$$

$$(1+y)^n = 1 + (n)(y) + \frac{1}{2!}(n)(n-1)(y)^2 + \frac{1}{3!}(n)(n-1)(n-2)(y)^3 + ..$$

No offence but you might have gone wrong... sub a small value of x like 0.00001 into the original and into your answer to check they are roughly the same!

Forming Differential Equations

"rate of change of V"

"is proportional to"

"is inversely proportional to"

Solving Differential Equations (Separation of Variables)

Be careful when moving things from one side to the other.

$$[\sin p]\frac{dp}{dt} = [3t^3]$$

$$\int[\sin p]\,dp = \int[3t^3]\,dt$$

$$-\cos p = \frac{3}{4}t^4 + c$$

$$p = \cos^{-1}\left(-\frac{3}{4}t^4 + c\right)$$

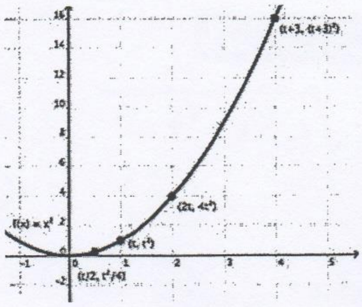
Connected Rates of Change

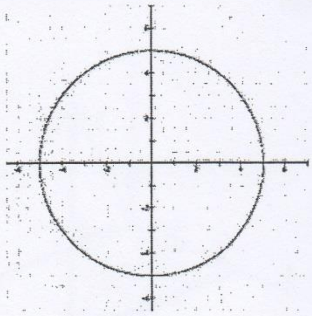
$$\frac{dV}{dt} = \frac{dV}{dx}\cdot\frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

Parametric Equations

$$\frac{dt}{dx} = \frac{1}{\frac{dx}{dt}}$$





x = t
y = t^2

Cartesian: $y = x^2$

x = $5\cos t$
y = $5\sin t$

Cartesian: $x^2 + y^2 = 5$

$\boxed{\text{Tangent/Normal: } m(x-a)=y-b}$

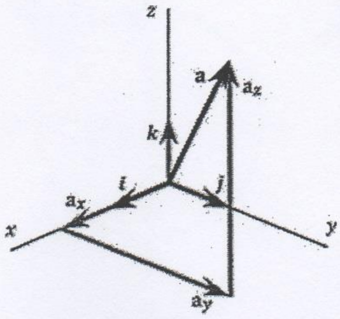
$$m = \frac{dy}{dx}$$

$$\int (y)\, dx = \int (y)\left(\frac{dx}{dt}\right) dt$$

$\boxed{\text{Areas:}}$

2D & 3D Vectors

$\underline{r} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$



UNDERLINE VECTORS!

Length: $\sqrt{a^2+b^2+c^2}$

$\underline{r} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ $\underline{s} = \begin{pmatrix} d \\ e \\ f \end{pmatrix}$

Dot Product: $\underline{r} \cdot \underline{s} = ad + be + cf$

$\underline{r} \cdot \underline{s} = \sqrt{a^2+b^2+c^2}\sqrt{d^2+e^2+f^2}\cos\theta$

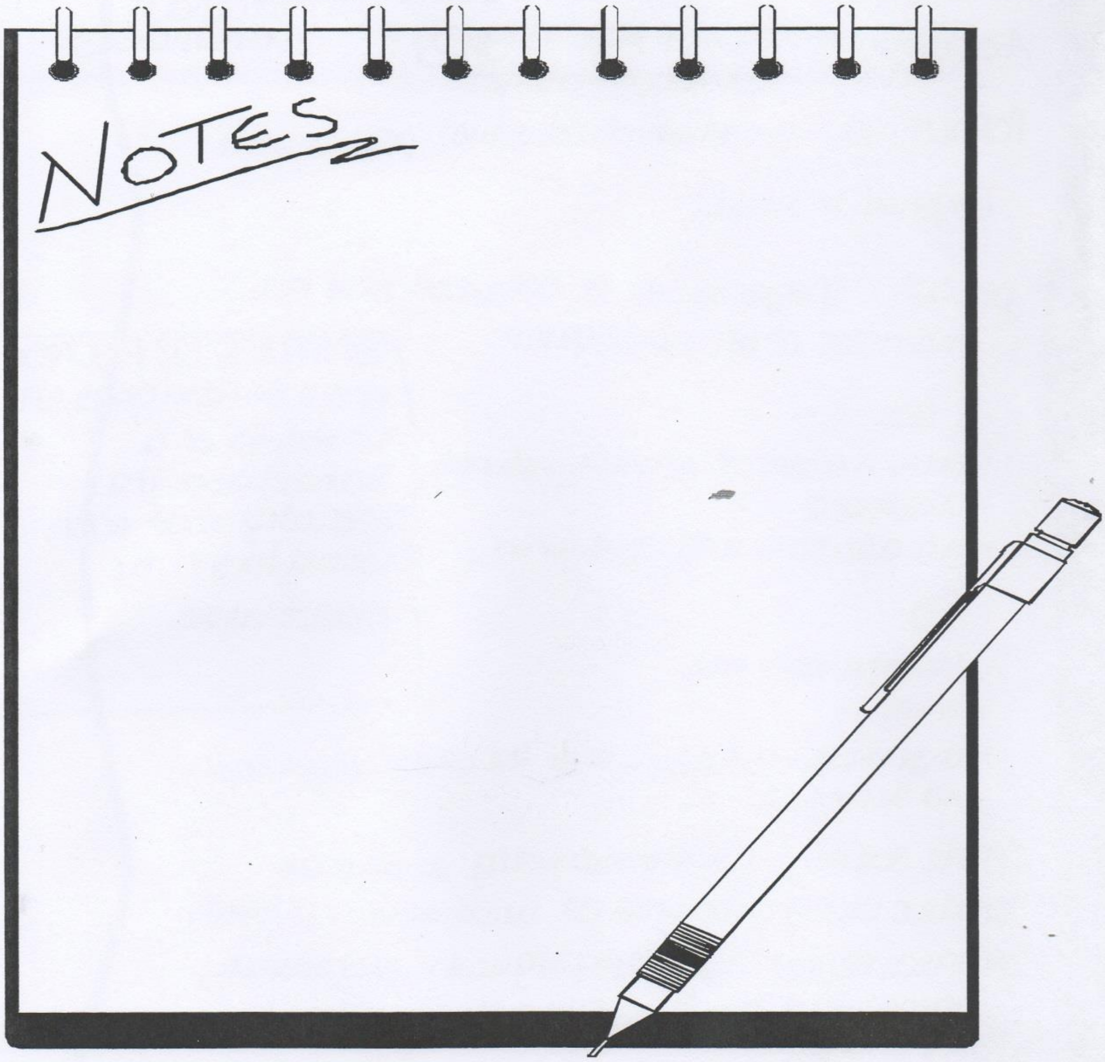
To prove vectors are perpendicular: $\underline{r} \cdot \underline{s} = 0$

If you know that vectors are perpendicular:

Vector Line Equation: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$
(go to the line) + (any amount)(direction of line)

$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

If in doubt draw a picture



Things from C2 which may be useful

Area of a triangle: $\frac{1}{2}ab\sin C$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$