C2 JANUARY 2011

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1. $$f(x) = x^4 + x^3 + 2x^2 + ax + b$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7.

(a) Show that $a + b = 3$. **(2)**

When $f(x)$ is divided by $(x + 2)$, the remainder is -8.

(b) Find the value of a and the value of b. **(5)**

a) $f(1) = 7 \Rightarrow 1 + 1 + 2 + a + b = 7$

$\therefore a + b = 3$ — (1)

b) $f(-2) = -8 \Rightarrow 16 - 8 + 8 - 2a + b = -8$

$\therefore -2a + b = -24$ — (2)

(1) − (2) $\Rightarrow 3a = 27$

$\therefore a = 9$

$\therefore b = -6$

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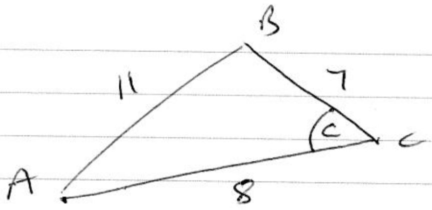
2. In the triangle ABC, $AB = 11$ cm, $BC = 7$ cm and $CA = 8$ cm.

(a) Find the size of angle C, giving your answer in radians to 3 significant figures.
(3)

(b) Find the area of triangle ABC, giving your answer in cm^2 to 3 significant figures.
(3)

a)



$$\cos C = \frac{7^2+8^2-11^2}{2\times7\times8} = -\frac{1}{14}$$

$\therefore C = 1.64^c$ (3 sf)

b) Area $= \frac{1}{2} \times 7 \times 8 \times \sin 1.64$

$= 27.9$ cm^2 (3 sf)

H 3 5 4 0 3 A 0 4 2 8

3. The second and fifth terms of a geometric series are 750 and –6 respectively.

Find

(a) the common ratio of the series, **(3)**

(b) the first term of the series, **(2)**

(c) the sum to infinity of the series. **(2)**

a) $u_n = ar^{n-1}$

$u_2 = 750$

$u_5 = -6$

$\therefore r^3 = \frac{-6}{750}$ $\therefore r = -0.2$

b) $u_2 = 750$

$\therefore u_1 = \frac{750}{-0.2} = -3750$

c) $S_\infty = \frac{a}{1-r} = \frac{-3750}{1.2} = -3125$

4.

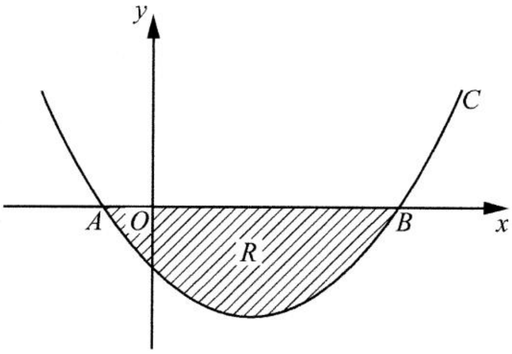


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x+1)(x-5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

a) $A(-1,0)$ $\quad B(5,0)$

b) $\int_{-1}^{5} (x+1)(x-5)\,dx$

$= \int_{-1}^{5} x^2 - 4x - 5 \; dx$

$= \left[\frac{1}{3}x^3 - 2x^2 - 5x\right]_{-1}^{5}$

$= \left(\frac{125}{3} - 50 - 25\right) - \left(-\frac{1}{3} - 2 + 5\right)$

$= -36$

negative sign shows area is below x-axis

$\therefore R = 36$

5. Given that $\binom{40}{4} = \frac{40!}{4!b!}$,

(a) write down the value of b.

(1)

In the binomial expansion of $(1+x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\frac{q}{p}$.

(3)

a) 36

b) coefficient of x^4 is $\frac{40!}{4!36!} = p$

coefficient of x^5 is $\frac{40!}{5!35!} = q$

$\therefore \frac{q}{p} = \frac{40!}{5!35!} \times \frac{4!36!}{40!} = \frac{36}{5} = 7.2$

6.
$$y = \frac{5}{3x^2 - 2}$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38	0.30	0.24	0.2

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2 - 2}\, dx$.

(4)

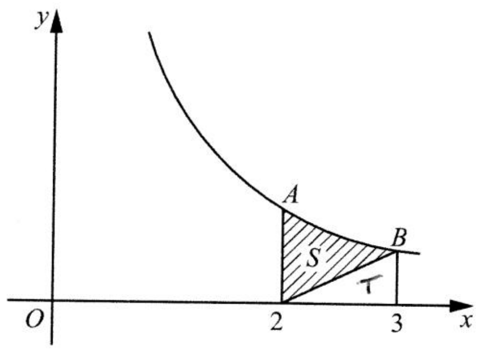


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points A and B on the curve, $x = 2$ and $x = 3$ respectively.

The region S is bounded by the curve, the straight line through B and (2, 0), and the line through A parallel to the y-axis. The region S is shown shaded in Figure 2.

(c) Use your answer to part (b) to find an approximate value for the area of S.

(3)

b) $\int_2^3 \frac{5}{3x^2-2}\, dx \approx \frac{1}{2} \times 0.25 (0.5 + 0.2 +$

$2(0.38 + 0.30 + 0.24))$

$= 0.125(0.7 + 1.84)$

$= 0.3175$

$= 0.32$ (2 d.p.)

Question 6 continued

c) $S = \int_2^3 \frac{5}{3x^2-2}\,dx$ - triangle T

area of T = $\frac{1}{2} \times 1 \times 0.2 = 0.1$

$\therefore S \approx 0.32 - 0.1 = 0.22$ (2 dp)

Q6

(Total 9 marks)

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7. (a) Show that the equation

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

can be written in the form

$$4\sin^2 x + 7\sin x + 3 = 0$$

(2)

(b) Hence solve, for $0 \leqslant x < 360°$,

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

(5)

a) $3\sin^2 x + 7\sin x = \cos^2 x - 4$

$\therefore \quad 3\sin^2 x + 7\sin x = 1 - \sin^2 x - 4$

$\therefore \quad 4\sin^2 x + 7\sin x + 3 = 0$

b) Let $y = \sin x$

$\therefore \quad 4y^2 + 7y + 3 = 0$

$7^2 - 4\times4\times3 = 1$ so it factorises

$(4y+3)(y+1) = 0$

$\therefore \quad 4y = -3$ or $y = -1$

i.e. $\sin x = -\frac{3}{4}$ or $\sin x = -1$

$\therefore \quad x = 228.6°, 311.4°, 270°$

8. (a) Sketch the graph of $y = 7^x$, $x \in \mathbb{R}$, showing the coordinates of any points at which the graph crosses the axes.

(2)

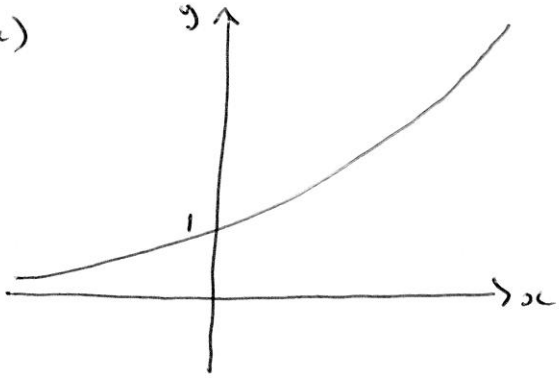
(b) Solve the equation

$$7^{2x} - 4(7^x) + 3 = 0$$

giving your answers to 2 decimal places where appropriate.

(6)

a)



b) Let $y = 7^x$

$\therefore\ y^2 - 4y + 3 = 0$

$\therefore\ (y-3)(y-1) = 0$

$\therefore\ y = 3$ or $y = 1$

i.e. $7^x = 3$ or $7^x = 1$

$\therefore\ x = \log_7 3$ or $x = \log_7 1$

$= \dfrac{\log_{10} 3}{\log_{10} 7}$ or $x = \dfrac{\log_{10} 1}{\log_{10} 7}$

$= 0.56$ or 0

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9. The points A and B have coordinates $(-2, 11)$ and $(8, 1)$ respectively.

Given that AB is a diameter of the circle C,

(a) show that the centre of C has coordinates $(3, 6)$,

(1)

(b) find an equation for C.

(4)

(c) Verify that the point $(10, 7)$ lies on C.

(1)

(d) Find an equation of the tangent to C at the point $(10, 7)$, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

a) centre is midpoint of $(-2,11)$ and $(8,1)$

which is at $\left(\frac{-2+8}{2}, \frac{11+1}{2}\right)$ i.e. $(3,6)$

b) radius is distance between $(3,6)$ and $(8,1)$

which is $\sqrt{5^2+5^2} = 5\sqrt{2}$

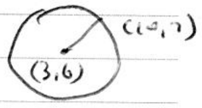
$\therefore$ equation is $(x-3)^2 + (y-6)^2 = 50$

c) when $x=10$, $y=7$

$LHS = 49 + 1 = 50 = RHS$

$\therefore$ $(10,7)$ lies on the circle)

d) gradient of radius $= \frac{1}{7}$



$\therefore$ gradient of tangent $= -7$

$\therefore$ equation is $y - 7 = -7(x-10)$

i.e. $y = -7x + 77$

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10. The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5-x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)

a) $V = 4x(25 - 10x + x^2)$

$= 100x - 40x^2 + 4x^3$

$\therefore \frac{dV}{dx} = 100 - 80x + 12x^2$

b) Maximum occurs at $\frac{dV}{dx} = 0$

$\therefore 100 - 80x + 12x^2 = 0$

$\therefore 3x^2 - 20x + 25 = 0$

$\therefore (3x - 5)(x - 5) = 0$

$\therefore x = \frac{5}{3}$ or $x = 5$

clearly $x = 5$ is a minimum since it gives $V = 0$

$\therefore V = 4 \times \frac{5}{3}(5 - 5/3)^2 = \frac{20}{3} \times \frac{100}{9} = 74\,2/27$

c) $\frac{d^2V}{dx^2} = -80 + 24x$

$x = 5/3 \Rightarrow \frac{d^2V}{dx^2} = -80 + 24 \times 5/3 = -40$

$\therefore$ This demonstrates $x = 5/3$ occurs at a maximum